Trading and Investment As A Science

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Speaker Profile

- Dr. Haksun Li
- CEO, Numerical Method Inc.
- (Ex-) Adjunct Professors, Advisor with the National University of Singapore, Nanyang Technological University, Fudan University, etc.
- Quantitative Trader/Analyst, BNPP, UBS
- PhD, Computer Sci, University of Michigan Ann Arbor
- M.S., Financial Mathematics, University of Chicago
- B.S., Mathematics, University of Chicago

Numerical Method Incorporated Limited

- A consulting firm in mathematical modeling, esp. quantitative trading or wealth management
- Products:
  - SuanShu
  - AlgoQuant
- Customers:
  - brokerage houses and funds all over the world
  - multinational corporations
  - very high net worth individuals
  - gambling groups
  - academic institutions

The Laymen's Approach

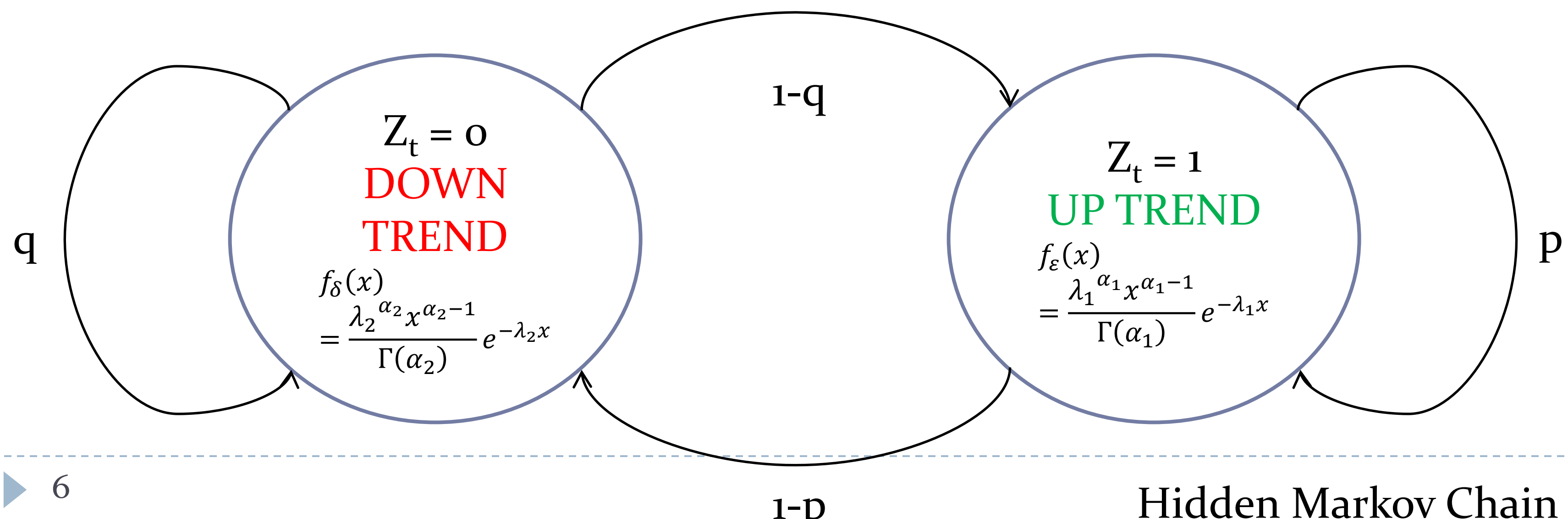
- Where does a trading idea come from?
  - Ex-colleagues
  - Hearsays
  - Newspaper
  - NOW TV, e.g., Moving Average Crossover (MA)
- Why is MA a good strategy?
- Out of a few thousand stocks, MA would almost be guaranteed to work on some.
  - It is very hard to come up with a strategy that never works on anything!

Not WHY, but WHEN.

- *The correct question to ask is: when is a strategy working?*
- How do we check these working conditions?
- How do we detect when the strategy stops working before we lose a lot of money?
- How much money are we expected to make? (How "working" is it?)
- How do we evaluate the risk(s)?

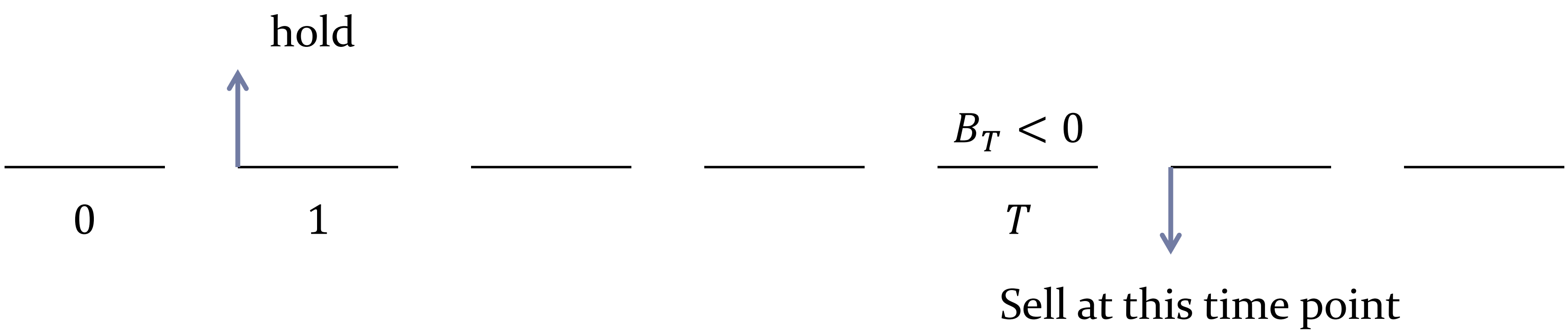
A Mathematical Analysis of Moving Average Crossover

- GMA(n, 1)
  - $B_t \geq 0 \text{ iff } P_t \geq \left(\prod_{j=0}^{n-1} P_{t-j}\right)^{\frac{1}{n}}$
    - $R_t \geq -\sum_{j=1}^{n-2} \frac{n-(j+1)}{n-1} R_{t-j}$ (by taking log)
  - $B_t < 0 \text{ iff } P_t < \left(\prod_{j=0}^{n-1} P_{t-j}\right)^{\frac{1}{n}}$
    - $R_t < -\sum_{j=1}^{n-2} \frac{n-(j+1)}{n-1} R_{t-j}$ (by taking log)



Hidden Markov Chain

Holding Time Distribution

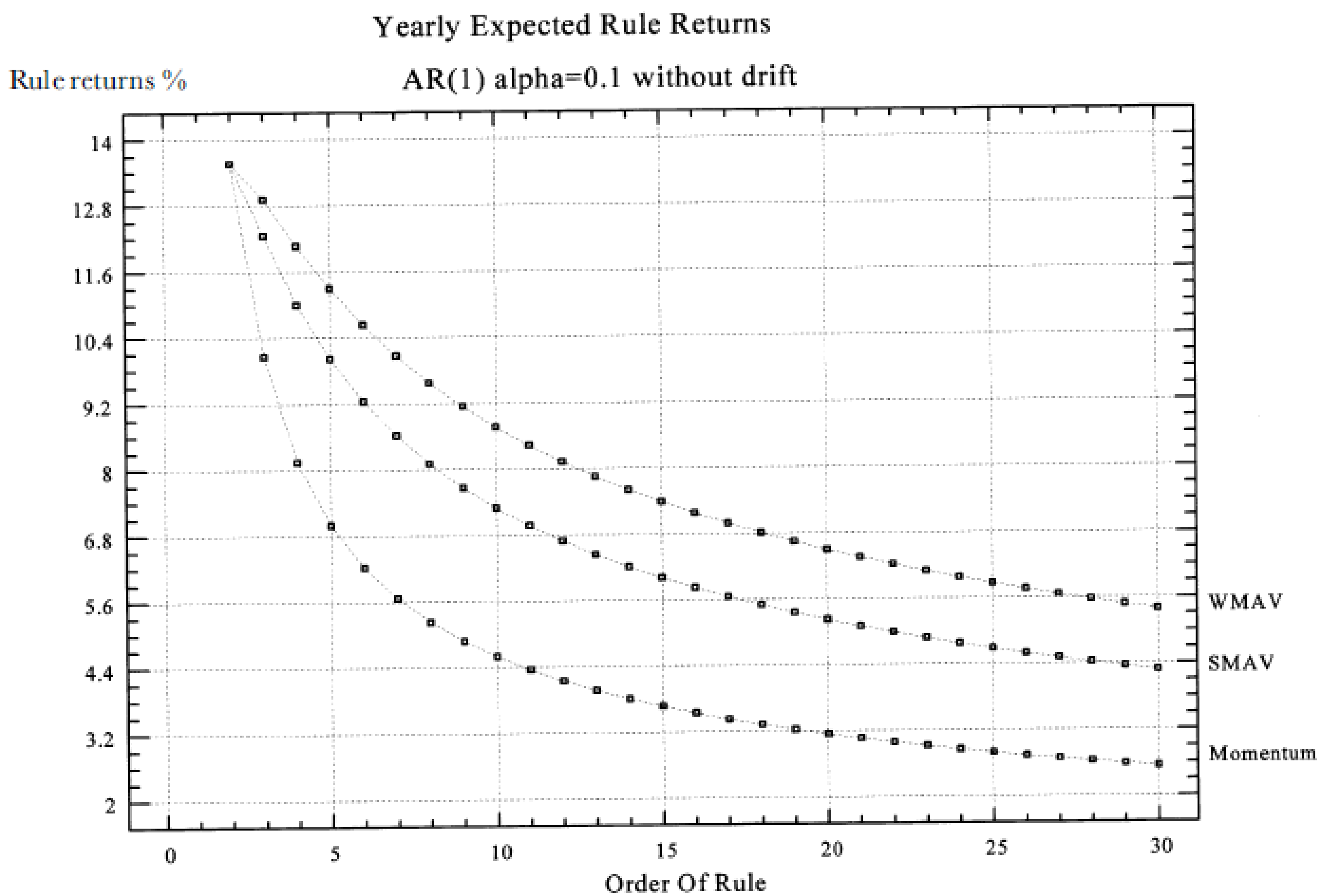


- $P(N = T)$
- $= P(B_T < 0, B_{T-1} \geq 0, \ldots, B_1 \geq 0, B_0 \geq 0)$
- $= P(Z_T = 0, Z_{T-1} = 1, \ldots, Z_1 = 1, Z_0 = 1)$
- $= P(Z_T = 0, Z_{T-1} = 1, \ldots, Z_1 = 1 | Z_0 = 1) P(Z_0 = 1)$
- $= \begin{cases} \Pi p^{T-1}(1-p), & \mathrm{T} \geq 1 \\ 1 - \Pi, & \mathrm{T} = 0 \end{cases}$

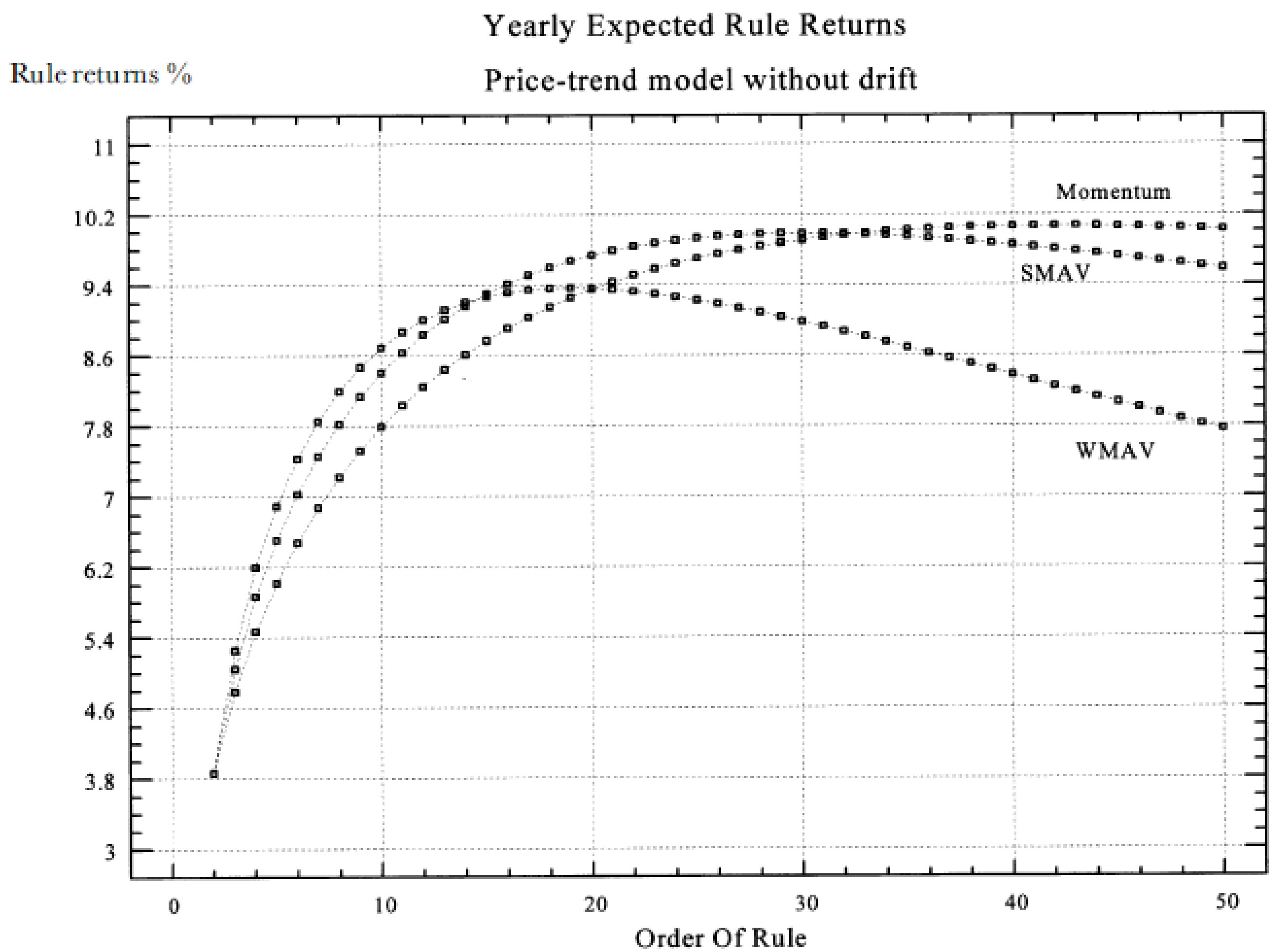
Expected Return

- $\Phi_{RR_T|N=T}(s) = \mathrm{E}\left[e^{\left\{i\left[\sum_{t=1}^{T} R_t \times I_{\{B_{t-1}\geq 0\}}\right]s\right\}}|N = T\right]$
- $\Phi_{RR_T}(s) = \sum_{T=0}^{\infty} \Phi_{RR_T|N=T}(s)P(N = T)$
- $\mathrm{E}(RR_T) = -i\Phi_{RR_T}{}'(0) = \frac{1}{1-p}\{\Pi p\mu_\varepsilon - (1-p)\mu_\delta\}$
- When is the expected return positive?
  - $\mu_\varepsilon \geq \frac{1-p}{\Pi p}\mu_\delta$, shock impact
  - $\mu_\varepsilon \gg \mu_\delta$, shock impact
  - $\Pi p \geq 1 - p$, if $\mu_\varepsilon \approx \mu_\delta$, persistence

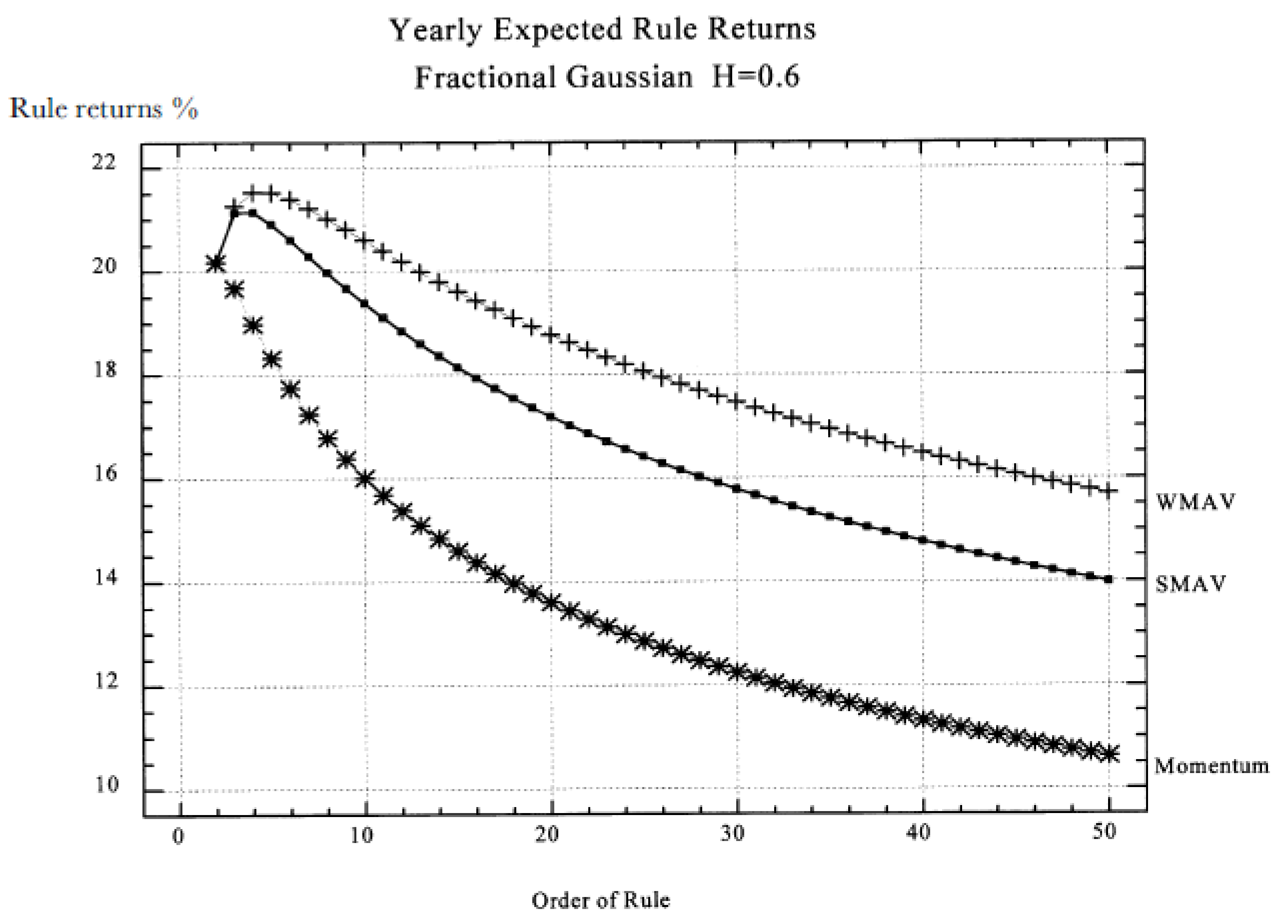
Monte Carlo: AR(1)



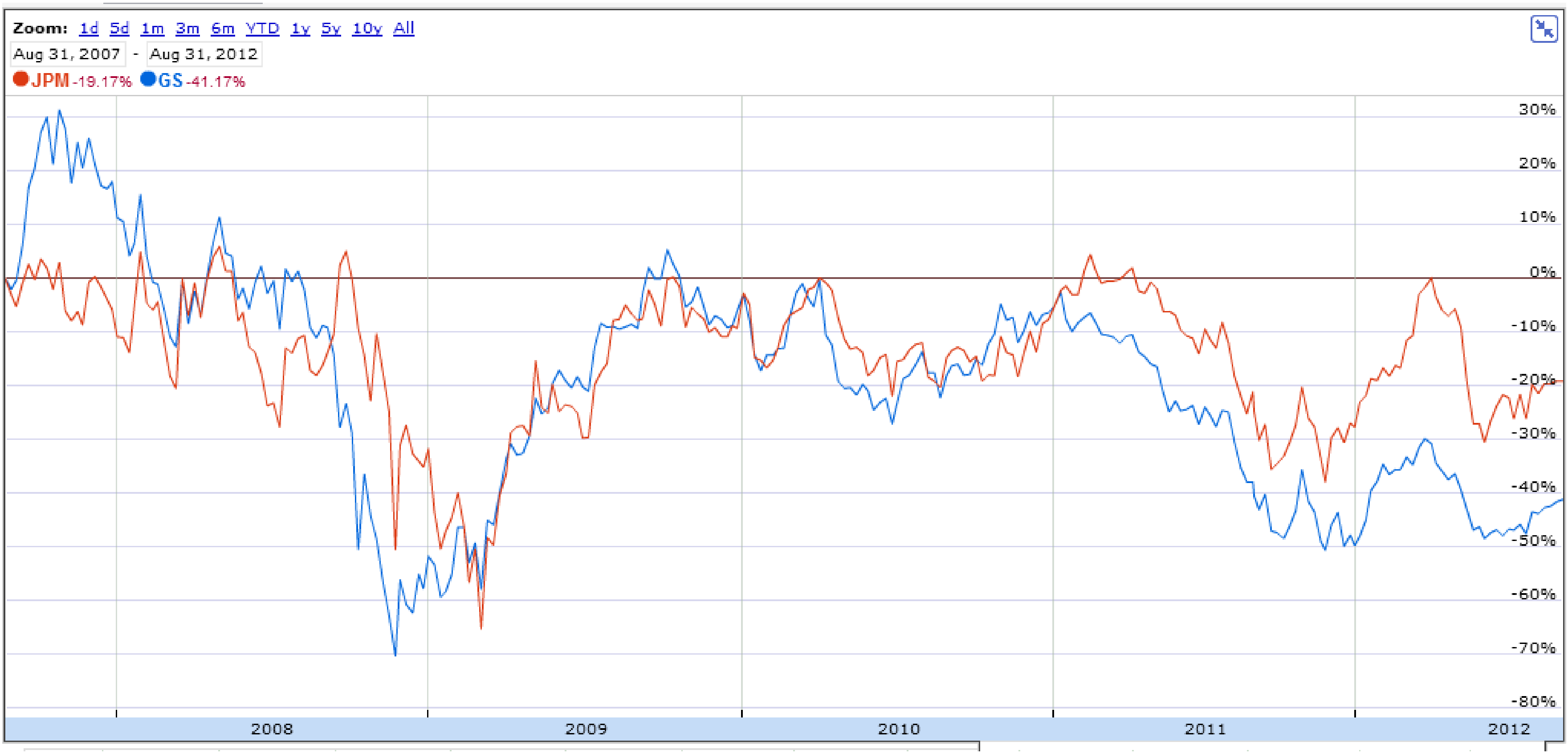
Monte Carlo: ARMA(1, 1)



Monte Carlo: ARIMA(0, d, 0)

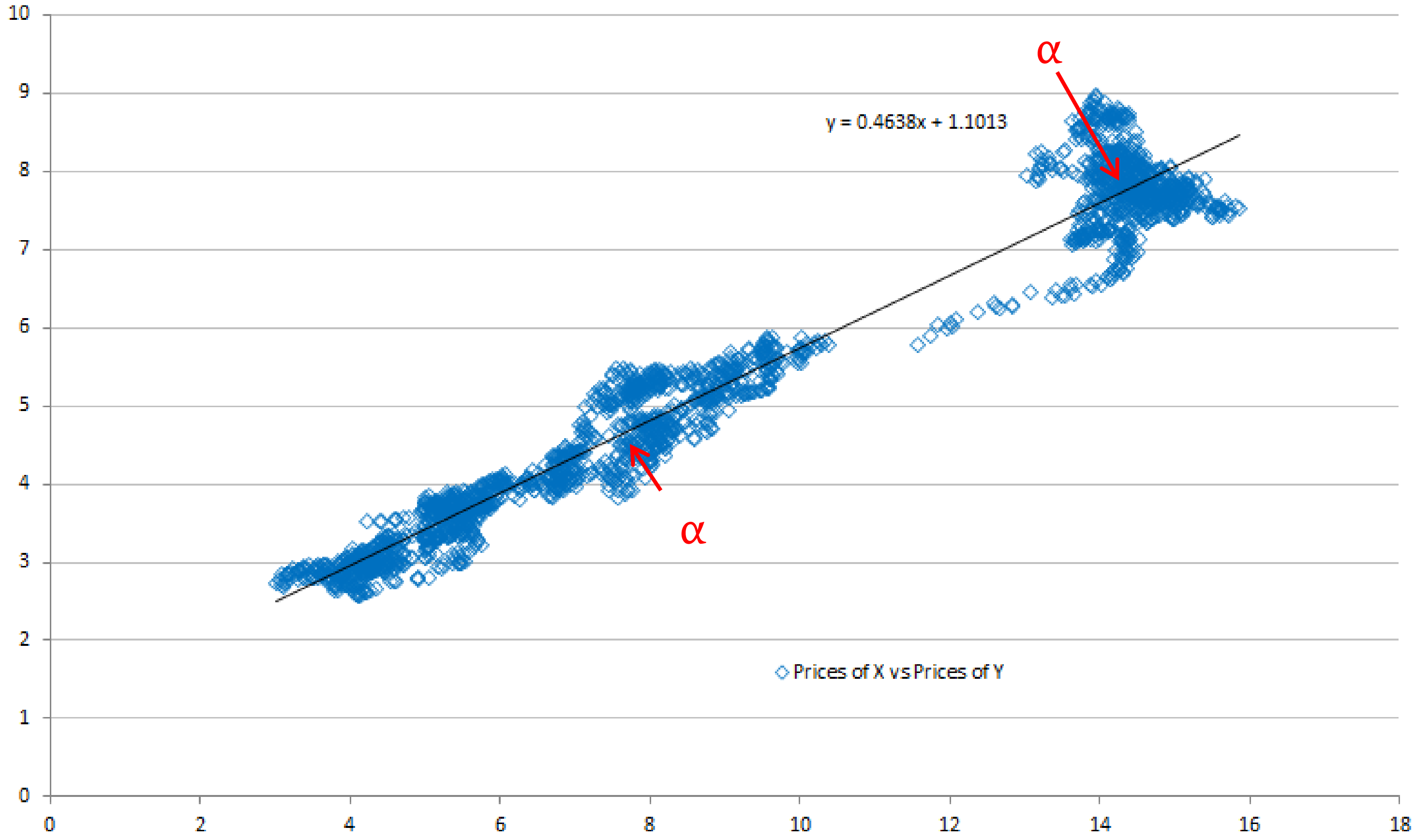


Pairs/Basket Trading

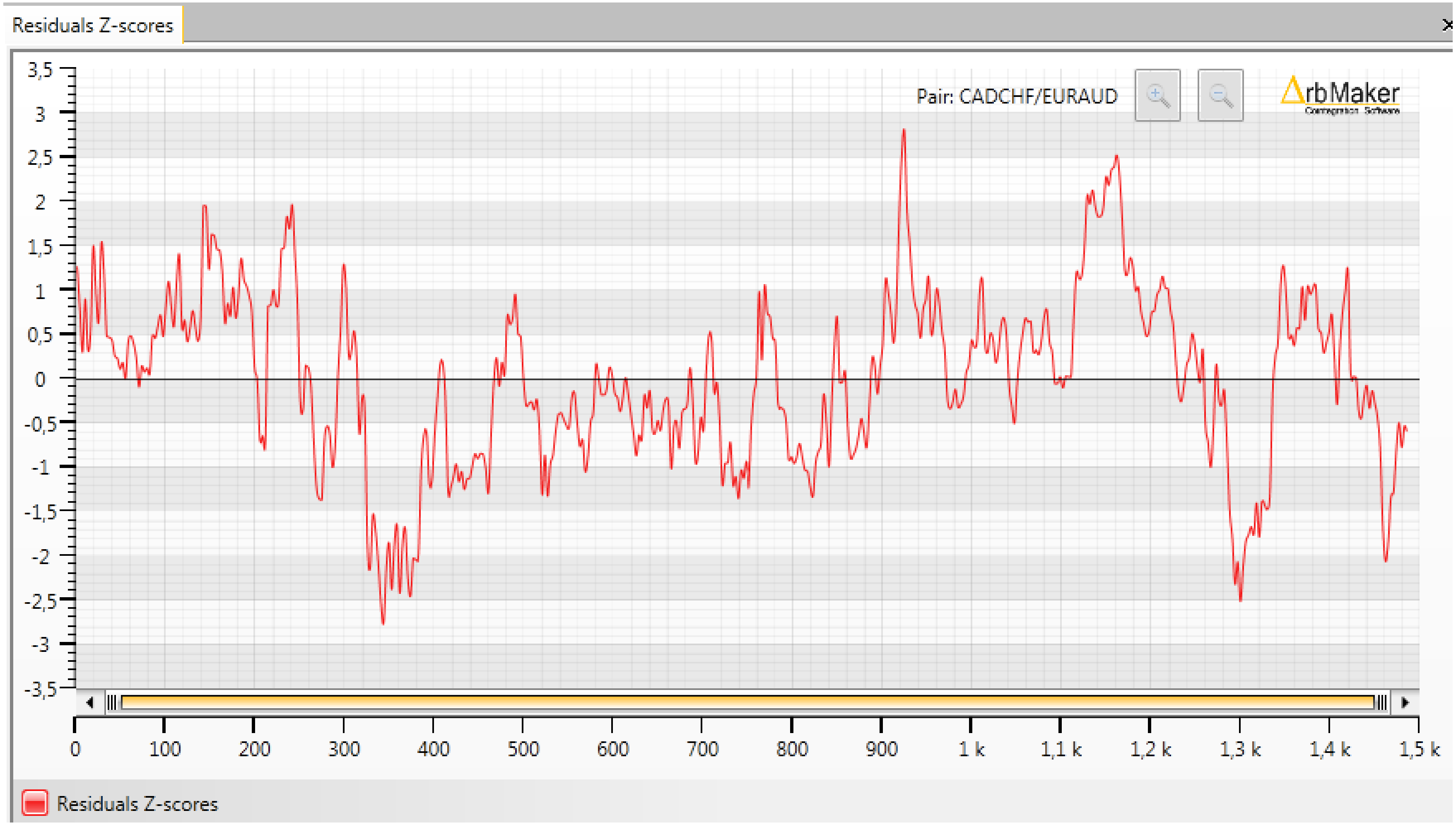


Cointegration

$$\Pi = \alpha\beta'$$



Mean Reversion



Hamilton-Jacobi-Bellman Equation

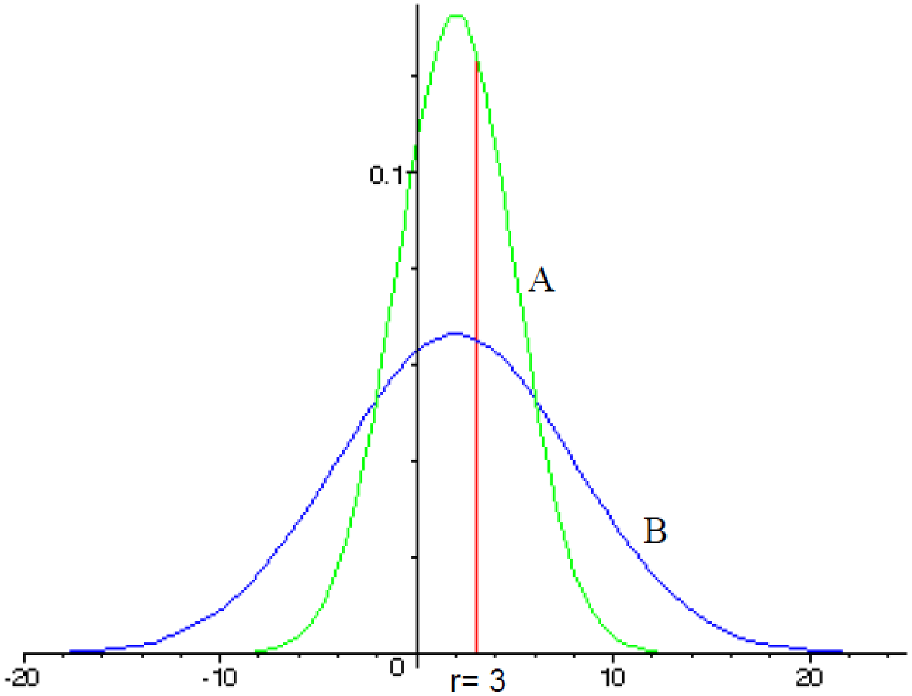
- $G(t, V_t, x_t) = \max_{h_t} E\{G(t, V_t, x_t) + \Delta G\}$
- Divide by time discretization, Δt.
- Take limit as $\Delta t \to 0$, hence Ito.
- $0 = \max_{h_t} E\{\Delta G\}$
- $\max_{h_t} E\left\{G_t dt + G_V(dV) + G_x(dx) + \frac{1}{2}G_{VV}(dV)^2 + \frac{1}{2}G_{xx}(dx)^2 + G_{Vx}(dV)(dx)\right\} = 0$
- $$\max_{h_t} E\left\{\begin{matrix} G_t dt + G_V(h_t k(\theta - x_t)dt + h_t \eta d\omega_t) + \\ G_x(k(\theta - x_t)dt + \eta d\omega_t) + \\ \frac{1}{2}G_{VV}(h_t k(\theta - x_t)dt + h_t \eta d\omega_t)^2 + \\ \frac{1}{2}G_{xx}(k(\theta - x_t)dt + \eta d\omega_t)^2 + \\ G_{Vx}(h_t k(\theta - x_t)dt + h_t \eta d\omega_t) \times (k(\theta - x_t)dt + \eta d\omega_t) \end{matrix}\right\} = 0$$
- The optimal portfolio position is ${h_t}^*$.

Best Mean Reversion Strategy

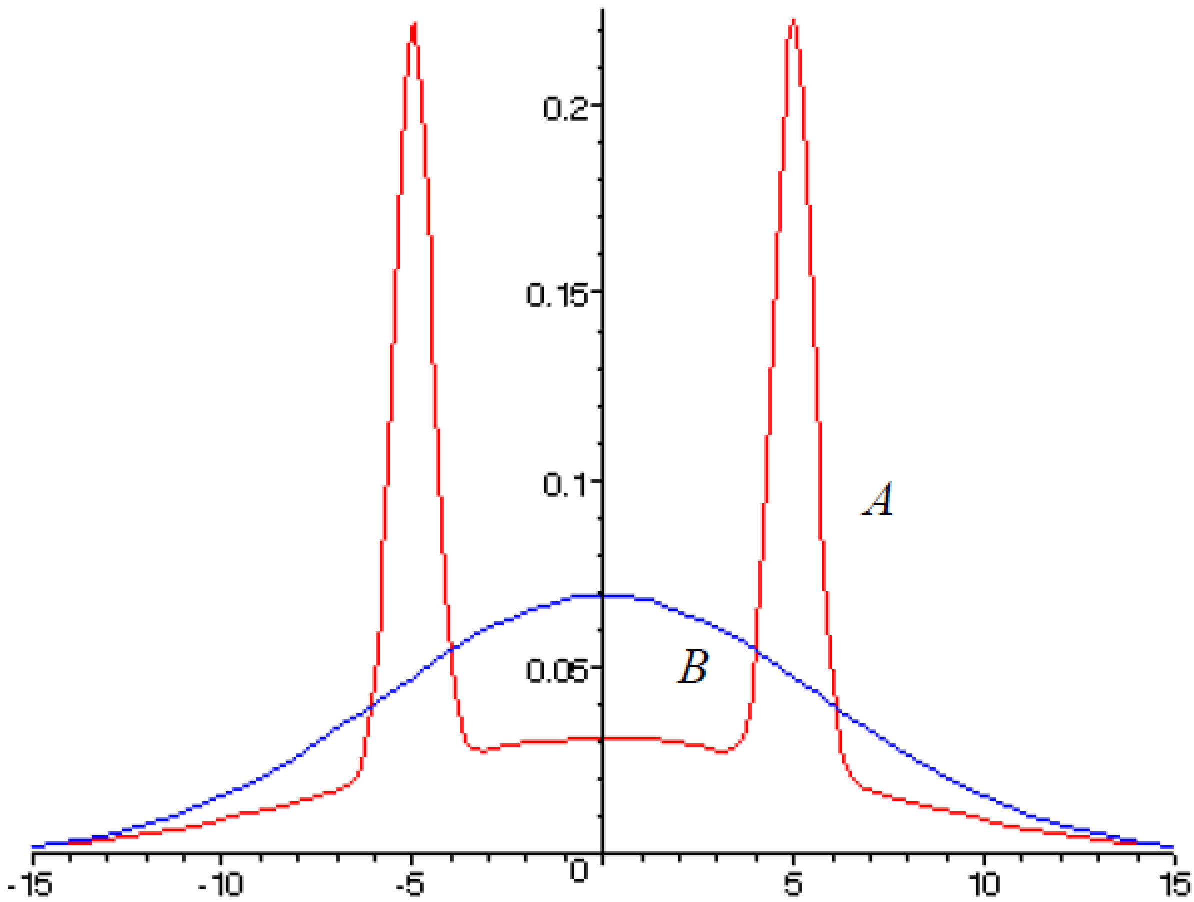
- $h(t)^* = \frac{V_t}{(1-\gamma)}\left[-\frac{k}{\eta^2}(x_t - \theta) + 2\alpha(t)x_t + \beta(t)\right]$
- $h(t)^* \sim -\frac{k}{\eta^2}(x_t - \theta)$

Portfolio Performance

- $\mathrm{sr}(x) = \frac{\mu(x) - r_f}{\sigma^2(x)} = \frac{x'E(r) - r_f}{x'Qx}$
- Markowitz Portfolio Optimization



A or B



Omega Measure

- To account for
  - the odds of wining and losing
  - the sizes of wining and losing
  - all moments of a return distribution
- We consider
  - $\Omega = \frac{E(r|r>L)\times P(r>L)}{E(r|r\leq L)\times P(r\leq L)}$
  - $\Omega = \frac{E(r|r>L)\left(1-F(L)\right)}{E(r|r\leq L)F(L)}$
- $\Omega = \frac{\int_{L}^{b=\max\{r\}}[1-F(r)]dr}{\int_{a=\min\{r\}}^{L} F(r)dr}$

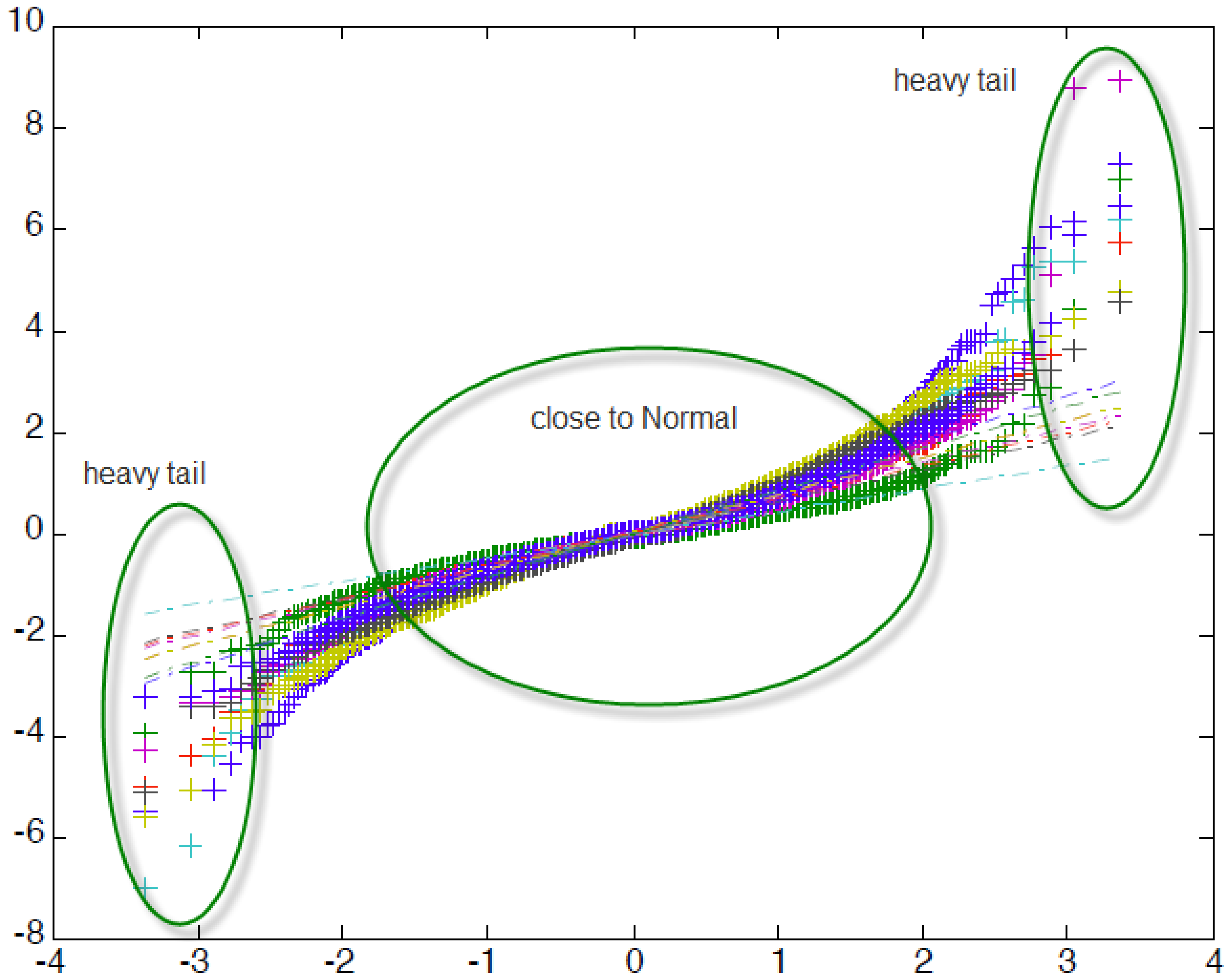
Portfolio Optimization

- Stochastic Portfolio Optimization
  - Unknown mean and covariance
  - http://numericalmethod.com/blog/2013/02/16/mean-variance-portfolio-optimization-when-means-and-covariances-are-unknown/
  - http://www.numericalmethod.com:8080/nmj2ee-war/faces/webdemo/lai2010.xhtml
- Second Order Conic Programming
  - Market impact constraints
- Polynomial Goal Programming
  - Optimize for the first four moments
- Differential Evolution
  - Non-convex, Non-linear, Integral optimization

Risk Management

- Given a loss distribution, F, quintile $1 > \mathrm{q} \geq 0.95$,
  - $\mathrm{VaR}_q = F^{-1}(q)$
- Suppose we hit a big loss, what is its expected size?
  - $\mathrm{ES}_q = E\left[X | X > \mathrm{VaR}_q\right]$
- VaR Computations
  - Historical Simulation
  - Variance-CoVariance
  - Monte Carlo simulation

QQ Plot

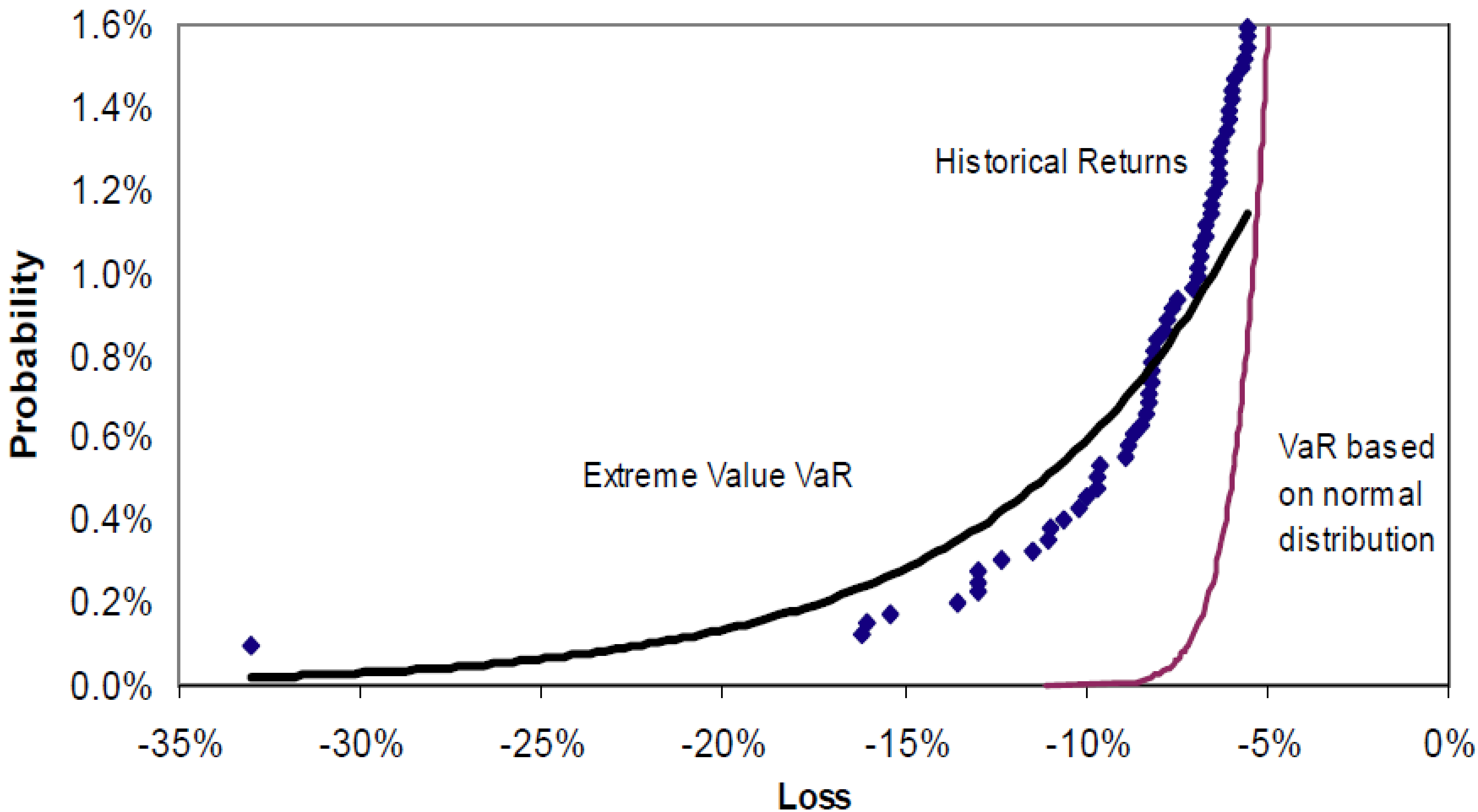


heavy tail
close to Normal
heavy tail
10
8
6
4
2
0
-2
-4
-6
-8
-4
-3
-2
-1
0
1
2
3
4

Extreme Value Theory

- Let $X_1, \ldots, X_n$ be i.i.d. with distribution $F(x)$.
- Let the sample maxima be $M_n = X_{(n)} = \max_i X_i$.
- $P(M_n \leq x) = P(X_1 \leq x, \ldots, X_n \leq x)$
- $= \prod_{i=1}^{n} P(X_i \leq x) = F^n(x)$
- What is $\lim_{n \to \infty} F^n(x)$?
  - $e^{-x^{-\alpha}} 1_{\{x>0\}}$

VaR Comparison



- http://www.fea.com/resources/pdf/a_evt_1.pdf