Beyond Markowitz Portfolio Optimization

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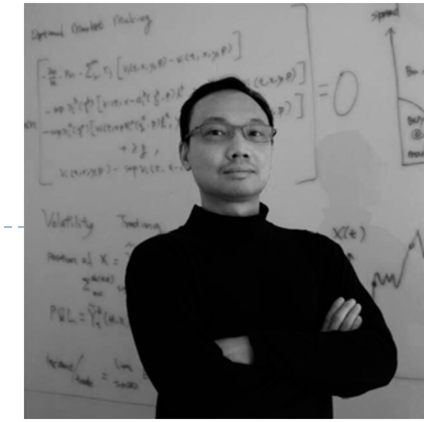
Speaker Profile



- Dr. Haksun Li
- CEO, [NM LTD.]
- Vice Dean, Fudan University ZhongZhi Big Data Finance and Investment Institute
- (Ex-)Adjunct Professors, Industry Fellow, Advisor, Consultant with the National University of Singapore, Nanyang Technological University, Fudan University, the Hong Kong University of Science and Technology
- Quantitative Trader/Analyst, BNPP, UBS
- Ph.D., Artificial Intelligence, University of Michigan, Ann Arbor
- M.S., Financial Mathematics, University of Chicago
- B.S., Mathematics, University of Chicago

Alpha Strategy in China

A Sample Alpha Strategy in China

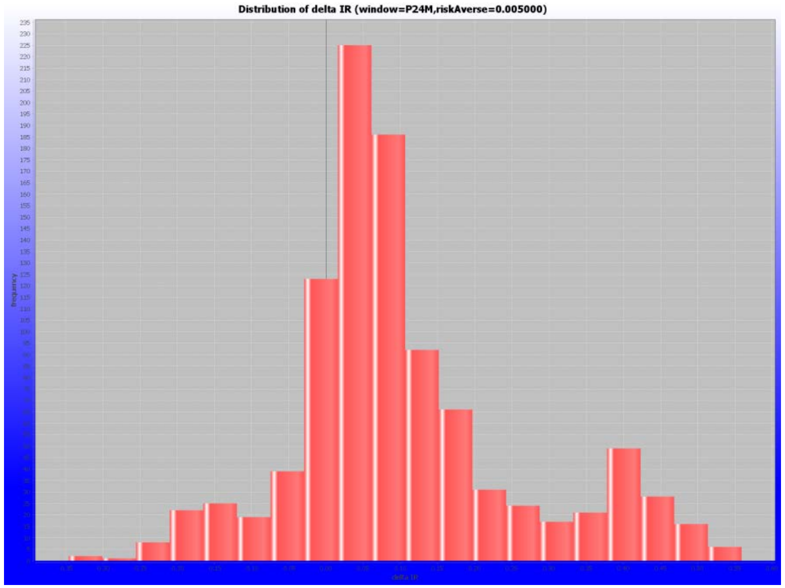
- Make clusters from 1000 factors
- Compute IR for each factor
- Weight for each factor in a cluster = IR_i / IR_total
- Score the stocks by sum of cluster values
- Sort stocks in each industry by scores
- Select top 20% in each industry
- Assign weight for each industry = weight in the market
- Assign weight for each stock = weight in the industry
- Hedge beta using CSI800

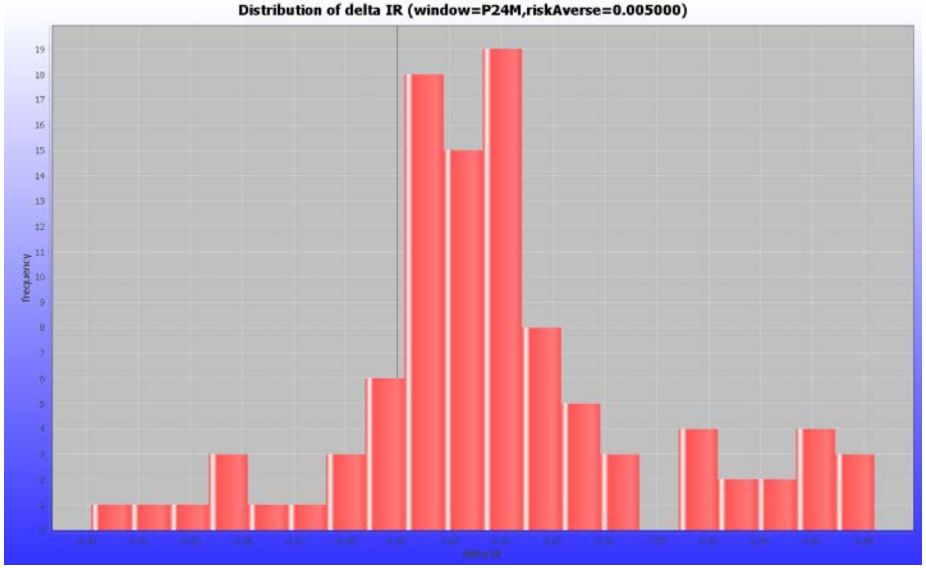
Problems with Chinese Alpha Strategies

- Reasons for failure of alpha strategies
  - Market characteristics change, e.g., big/small firm factor no longer effective
  - Futures backwardation, difference unpredictable
- Most alpha strategies are more or less the same
  - Similar pools of factors
  - Similar ways of assigning weights
- Factors used mainly as a way to do filtering
  - No prediction
- No mathematical models
  - Only sets of ad hoc heuristics

Solutions – Optimize Capital Allocation

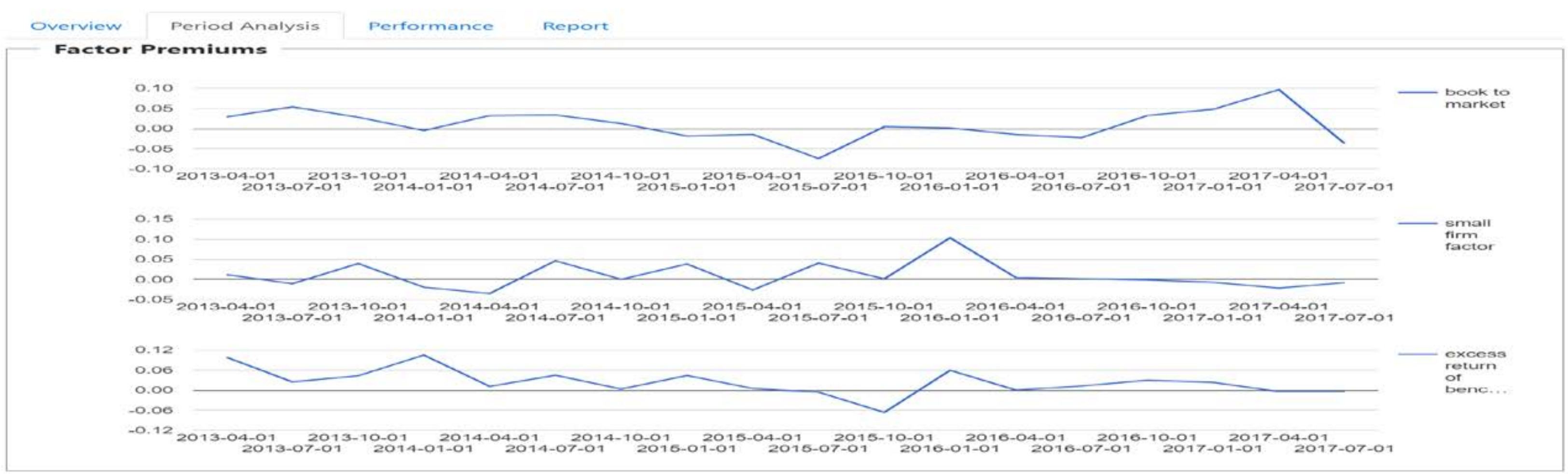
- Given the same set of stocks to long, different weightings give different P&Ls





Solutions – Predictive Factor Model

- We can build mathematical predictive models using factors
  - The model predicts expected returns of stocks
  - No longer used just as a filter
- Can scientifically evaluate the usefulness, robustness and the time-dependent characteristics of factors



Problems with Markowitz Portfolio Optimization

Why Portfolio Optimization

- FoF asset allocation
  - How much capital to assign to each fund?
- Portfolio asset allocation
  - How much capital to assign to each strategy?
- Alpha strategy asset allocation
  - How much capital to assign to each stock?

Harry Markowitz

- It all starts with Markowitz in 1952…
  - Standard textbook model
  - Widely taught in universities
  - MBA courses
- Won the Nobel Memorial Prize in Economic Sciences in 1990.

Modern Portfolio Theory – Insights

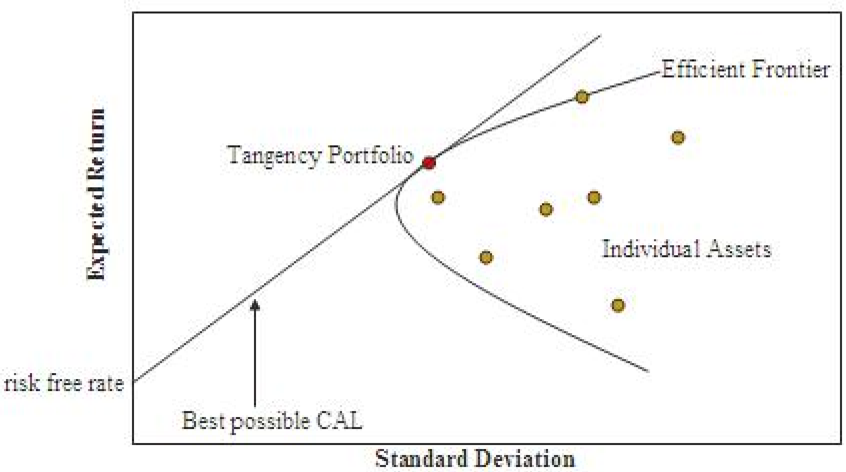
- An asset's risk and return should be assessed by how it contributes to a portfolio's overall risk and return, but not by itself.
- Mean-Variance (MV) optimization
  - Investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one.
  - An investor who wants higher expected returns must accept more risk.
  - An investor can have individual risk aversion characteristics in terms of the risk (tolerance) parameter.

Modern Portfolio Theory – Math

- $\max_{\omega}\{\omega \, \mathrm{E}(r_{t+1}) - \lambda\omega'\Sigma_t\omega\}$
  - where ω is the optimal portfolio weights
  - $\mathrm{E}(r_{t+1})$ is the expected return for the **next** period
  - Σ_t is the covariance matrix for the assets
- Constraints: $A\omega \le b$
  - No short selling: $-\mathrm{I}\omega \le 0$
- Alternatively, we have
  - $\min_{\omega}\{\omega'\Sigma_t\omega - \lambda\omega \, \mathrm{E}(r_{t+1})\}$
- Solution: Quadratic Programming
- NM:
  - http://redmine.numericalmethod.com/projects/public/repository/svn-algoquant/show/core/src/main/java/com/numericalmethod/algoquant/model/portfoliooptimization/markowitz

Efficient Frontier

- Given
  - $\omega\,\mathrm{E}(r_{t+1}) = \mu$
- Find ω s.t.,
  - $\omega_{eff} = \underset{\omega}{\operatorname{argmin}}\{\lambda\omega'\Sigma_t\omega\}$



Markowitz's Theory的问题

- Require the knowledge of means and covariances.
  - Too many parameters to estimate: $N + \frac{N^2+N}{2}$.
    - For $N = 300$, we have 45,450 parameters to estimate.
    - For $N = 3000$, we have 4,504,500 parameters to estimate.
    - Chopra & Ziemba (1993) shows that errors in means are about 10x as important as errors in variances, and errors in variances are about 2x important as errors in covariances.
  - Time varying. Tied to business cycles.

Problems with Sample Covariance Matrix

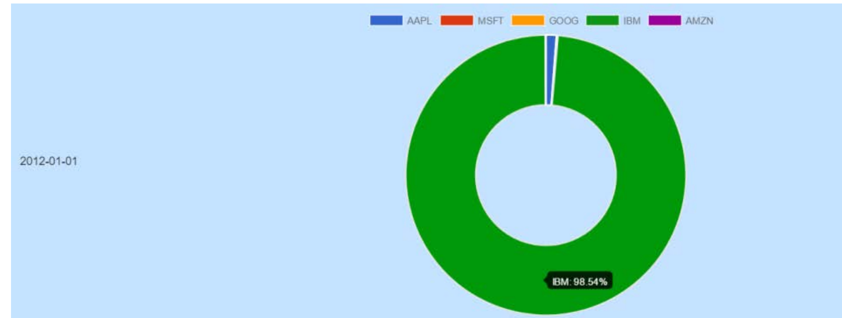
- A sample covariance matrix is often ill-conditioned, nearly singular, sometimes not even invertible and sometimes not even positive semi-definite.
  - dimension: p, number of samples: n
  - $\frac{p}{n} > 1$, matrix not invertible
  - $\frac{p}{n} < 1$ but not negligible, matrix ill-conditioned
- Linear dependency among stocks.
  - Asynchronous data
  - incomplete data
  - artificial changes due to stress-tests
- Error Maximization:
  - Largest sample eigenvalues are systematically biased upwards.
  - Smallest sample eigenvalues are systematically biased downwards.
  - Inverting a sample covariance matrix increases significantly the estimation error.
  - Capital allocated to the extreme eigenvalues where they are most unreliable.

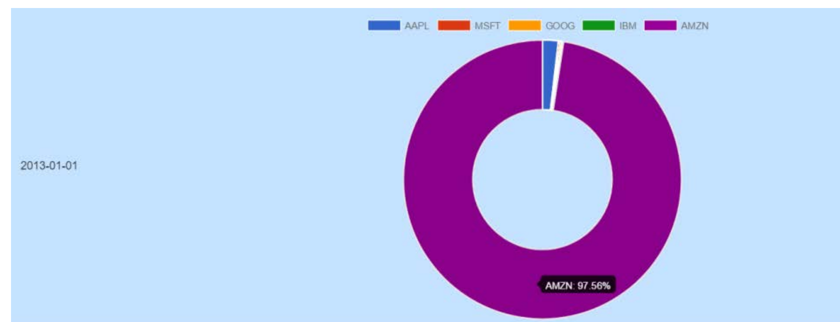
Problems with Sample Mean

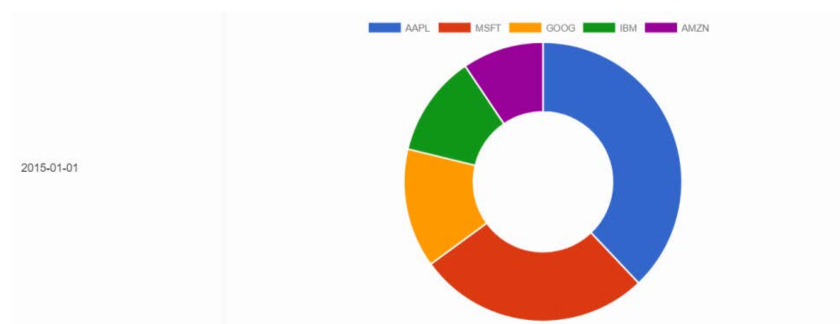
- Sample mean is only an estimation using TWO data points, namely the TWO end points, regardless of how big the sample size is.
- Given a set of historical returns $\{r_1, \dots, r_t\}$, the sample mean is
- $\bar{r} = \sum_{i=1}^{t} r_i$
- $\approx \sum_{i=1}^{t} \log(1 + r_i) = \sum_{i=1}^{t} \log(p_i) - \log(p_{i-1})$
- $= \log(p_t) - \log(p_0)$
- Assume returns follow Gaussian distribution.
  - Nassim Nicholas Taleb: After the stock market crash (in 1987), they rewarded two theoreticians, Harry Markowitz and William Sharpe, who built beautifully Platonic models on a Gaussian base, contributing to what is called Modern Portfolio Theory. Simply, if you remove their Gaussian assumptions and treat prices as scalable, you are left with hot air. The Nobel Committee could have tested the Sharpe and Markowitz models—they work like quack remedies sold on the Internet—but nobody in Stockholm seems to have thought about it.

Problems with Diversification

- Litterman & et al. (1992, 1999, 2003):
  - When unconstrained, portfolios will have large long and short positions.
  - When subject to long only constraint, capital is allocated only to a few assets.
- Best & Grauer (1991): a small increase in expected return can consume half of the capital.





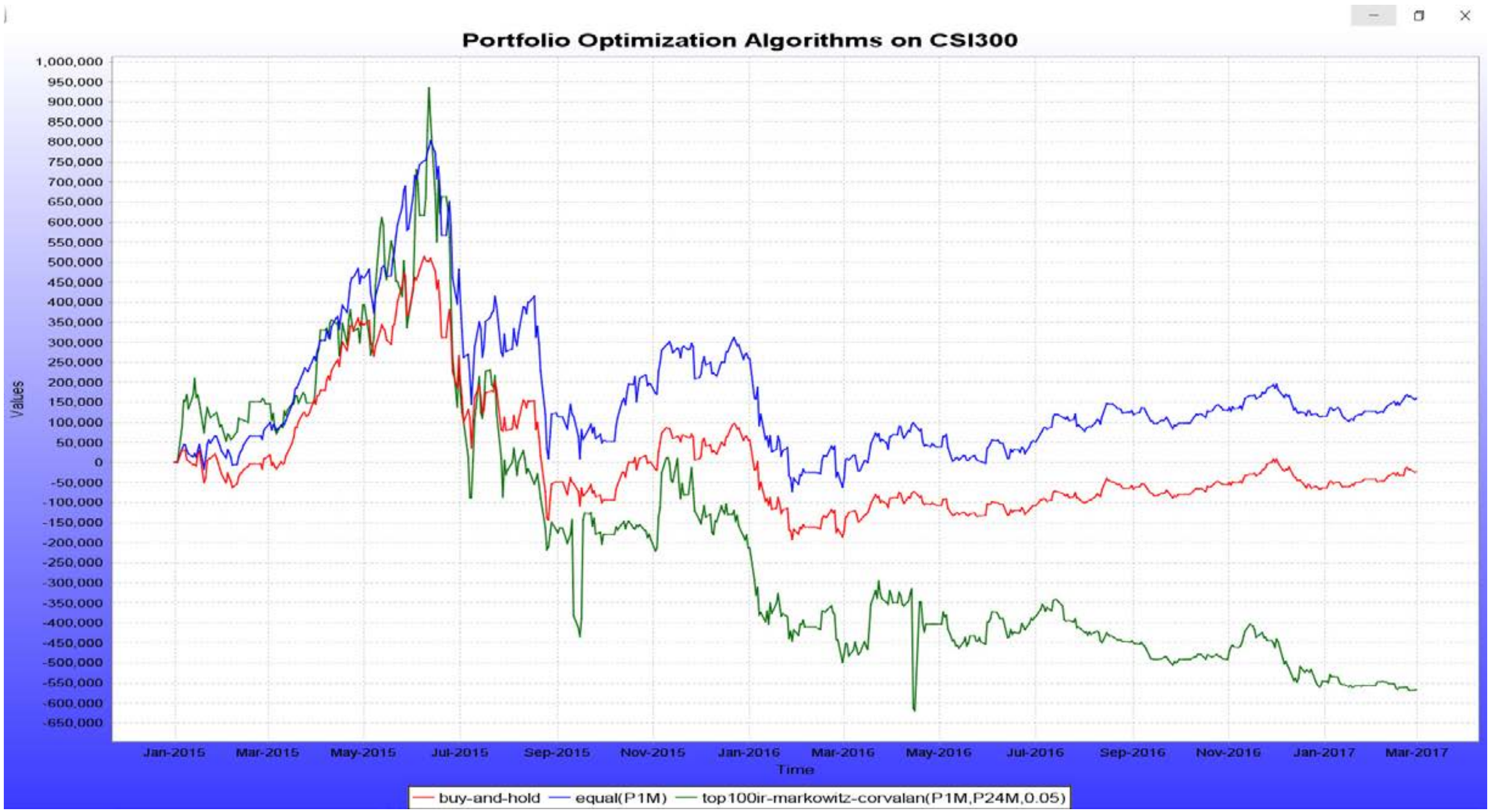


Problems with Constraints

- Minimizing variance
  - $\max_{\omega} \omega\, \mathrm{E}(r_{t+1})$, s.t.,
  - $\omega' \Sigma_t \omega \leq \sigma_{MAX}$
  - $1'\omega = 1$
  - $\omega \geq 0$
- Market impact
  - $\max_{\omega} \left\{ \omega\, \mathrm{E}(r_{t+1}) - \lambda_P \omega' \Sigma_t \omega - \lambda_M \sum_{j=1}^{n} \left(m_j |\omega_j|^{\frac{3}{2}} + c_j |\omega_j| \right) \right\}$
- Diversification constraints (sector exposure)
  - $\sum_{j \in S_i} |\omega_j^0 + \omega_j| \leq u_i$ for sector $i = 1, \ldots, S$
- Tax, transaction costs, etc.

Problem with Performance

- P&L often worse than the 1/N strategy (equal weighting).



Comments on Markowitz

- Wesley Gray: Although Markowitz did win a Nobel Prize, and this was partly based on his elegant mathematical solution to identifying mean-variance efficient portfolios, a funny thing happened when his ideas were applied in the real world: mean-variance performed poorly. The fact that a Nobel-Prize winning idea translated into a no-value-add-situation for investors is something to keep in mind when considering any optimization method for asset allocation …complexity does not equal value!

Solutions for Practical Portfolio Optimization

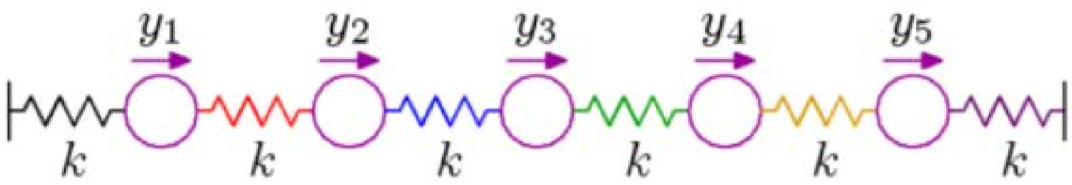
Solutions to Estimating Covariance – Dimension Reduction

- Dimension reduction via multifactor models
  - Relate the *i*-th asset returns r_i to k factors $f_1, \ldots, f_k$ by
  - $r_i = \alpha_i + (f_1, \ldots, f_k)'\beta_i + \epsilon_i$
  - α_i, β_i are unknown regression parameters; ϵ_i unobserved random noise with mean 0 and are uncorrelated.
  - $\mathrm{Cov}\left(r_{it}, r_{jt}\right) = \beta_{it}' \, \mathrm{V}(f) \, \beta_{jt}' + \mathrm{Cov}\left(\epsilon_{it}, \epsilon_{jt}\right)$
  - E.g., alpha strategy, Fama-French model, CAPM, APT
  - NM:
    - http://redmine.numericalmethod.com/projects/public/repository/svn-algoquant/show/core/src/main/java/com/numericalmethod/algoquant/model/factormodel

Solutions to Estimating Covariance – Shrinkage Estimators

- Pull the extreme eigenvalues back to the mean.
- Ledoit and Wolf (2003, 2004):
  - $\hat{\Sigma} = \hat{\delta}\hat{F} + \left(1 - \hat{\delta}\right)S$
    - $\hat{\delta}$ is an estimator of the optimal shrinkage constant
    - $\hat{F}$ is given by mean of the prior distribution or a structured covariance matrix, which has much fewer parameters than $N + \frac{N^2+N}{2}$.
    - S the sample covariance
  - NM:
    - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/stats/descriptive/covariance/LedoitWolf2004.html
    - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/model/returns/moments/MomentsEstimatorLedoitWolf.html
- Ledoit and Wolf (2012): nonlinear shrinkage

Inverse Covariance Matrix vs Covariance Matrix



inverse-covariance matrix or covariance matrix?

$$\mathbf{K}^{-1} = \frac{k}{T}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \qquad \mathbf{K} = \frac{T}{k}\begin{bmatrix} 0.83 & 0.67 & 0.50 & 0.33 & 0.17 \\ 0.67 & 1.33 & 1.00 & 0.67 & 0.33 \\ 0.50 & 1.00 & 1.50 & 1.00 & 0.50 \\ 0.33 & 0.67 & 1.00 & 1.33 & 0.67 \\ 0.17 & 0.33 & 0.50 & 0.67 & 0.83 \end{bmatrix}$$

Solutions to Estimating Covariance – Covariance Selection

- Dempster (1972): the covariance structure of a multivariate normal population can be simplified by setting elements of the inverse of the covariance matrix to zero.
- Awoye, OA; (2016): Graphical LASSO
- NM:
  - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/model/covarianceselection/lasso/CovarianceSelectionGLASSOFAST.html
  - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/model/covarianceselection/lasso/CovarianceSelectionLASSO.html

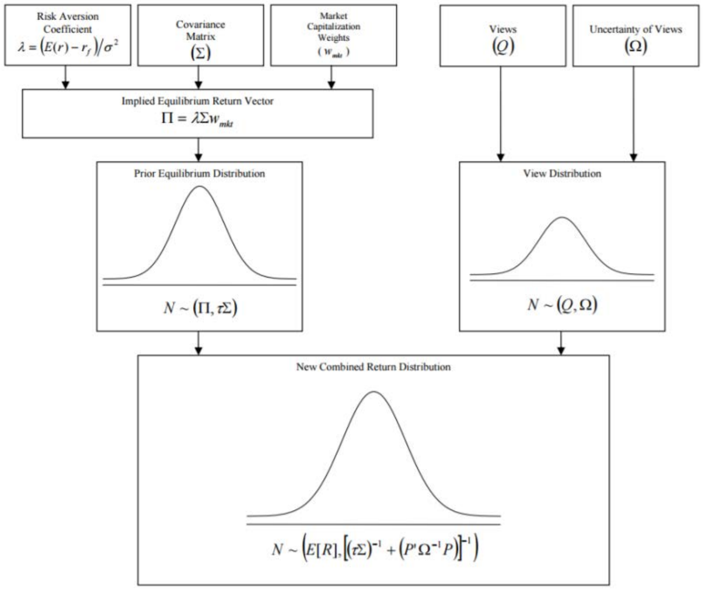
Solutions to Estimating Covariance – Nearest Positive Definite Matrix

- Matrix made Positive Definite
  - Goldfeld, Quandt and Trotter
  - Matthews and Davies
  - Positive diagonal
  - NM:
    - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/algebra/linear/matrix/doubles/operation/positivedefinite/package-summary.html
- Nearest Covariance/Correlation Matrix
  - Nicholas J. Higham (1988, 2013)
  - Defeng, Sun (2011, 2006)

Solutions to Estimating Mean – Statistical Methods

- Trading signals
  - NM:
    - http://redmine.numericalmethod.com/projects/public/repository/svn-algoquant/show/core/src/main/java/com/numericalmethod/algoquant/model
- Multifactor models: $r_i = \alpha_i + (f_1, \dots, f_k)'\beta_i + \epsilon_i$
- Shrinkage

Solutions to Estimating Mean – Black-Litterman



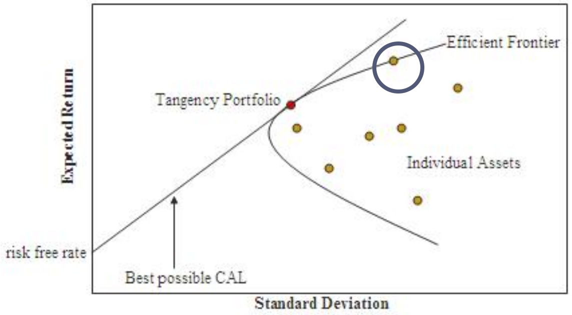
- Combined Return Vector
  - $\mathrm{E}(R) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]$
    - P: a matrix that identifies the assets involved in the views $(K \times N)$
    - Ω: a diagonal covariance matrix of error terms from the expressed views representing the uncertainties in each view $(K \times K)$
    - Π: the implied equilibrium return vector $(N \times 1)$
    - Q: the view vector $(K \times 1)$

Solutions to Diversification – Using Constraints

- Black-Litterman
- Diversification constraints, e.g.,
  - lower and upper bounds
  - sector exposure

Solutions to Diversification – Almost Efficient Portfolios

- MVO intends to give an optimized portfolio in terms of risk-reward
- MVO does not intend to give a diversified portfolio
- Many portfolios on the efficient frontier are indeed concentrated
- However, there are many well diversified portfolios within a small neighborhood of the efficient frontier
- Almost Efficient Portfolios:
  - $\max_{\omega} D(\omega)$ s.t., (D is the diversification criterion.)
    - $\sqrt{\omega'\Sigma\omega} \le \sigma^{\text{eff}} + \Delta\sigma$, relaxation of portfolio variance
    - $R^{\text{eff}} - \Delta R \le \omega' r$, relaxation of portfolio expected return
    - $1'\omega = 1$



- NM:
  - http://numericalmethod.com/blog/2013/06/19/solving-the-corner-solution-problem-of-portfolio-optimization/
  - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/model/corvalan2005/diversification/package-summary.html

Second Order Conic Programming

- $\min_x f'x$, s.t.,
  - $\|A_i x + b_i\|_2 \le c_i' x + d_i,\ i = 1, \dots, m$
  - $Fx = g$
- LP, QP
- Solution: interior point method

Solutions to Imposing Constraints – Second Order Conic Programming

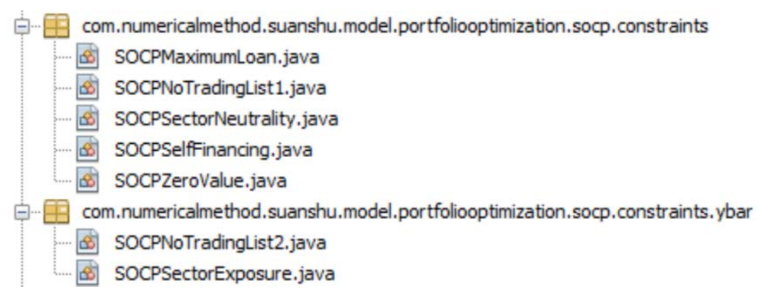
- Market impact
  - $\sum_{j=1}^{n}\left(m_j|\omega_j|^{\frac{3}{2}}\right) \le t_2$

$$||0||_2 \le t_2 - \sum_{j=1}^{n}\beta_j,$$
$$||0||_2 \le \bar{x}_j - (y_j - w_j^0),\ j = 1, \cdots, n,$$
$$||0||_2 \le \bar{x}_j - (-y_j + w_j^0),\ j = 1, \cdots, n,$$
$$||\begin{pmatrix} \bar{x}_j \\ \frac{\beta_j}{m_j} - \frac{s_j}{2} \end{pmatrix}||_2 \le \frac{\beta_j}{m_j} + \frac{s_j}{2},\ j = 1, \cdots, n,$$
$$||\begin{pmatrix} s_j \\ \frac{1-\bar{x}_j}{2} \end{pmatrix}||_2 \le \frac{1+\bar{x}_j}{2},\ j = 1, \cdots, n.$$

- Diversification constraints (sector exposure)
  - $\sum_{j\in S_i}|\omega_j^0 + \omega_j| \le u_i$ for sector $i = 1, \ldots, S$

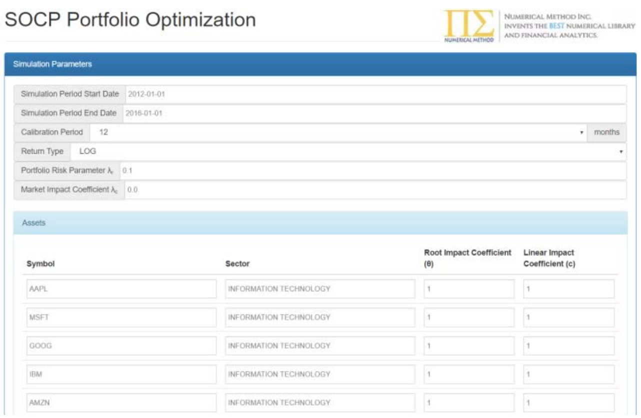
$$||0||_2 \le -\sum_{j\in S_i}\bar{y}_j + u_i \Longleftrightarrow ||A_i^\top z + C_i||_2 \le b_i^\top z + d_i, \quad i = 1, \cdots, k$$
$$A_i^\top = 0_{1\times n},\ C_i = 0,\ b_i = -\sum_{j\in S_i} e_j,\ d_i = u_i,\ z = \bar{y},$$

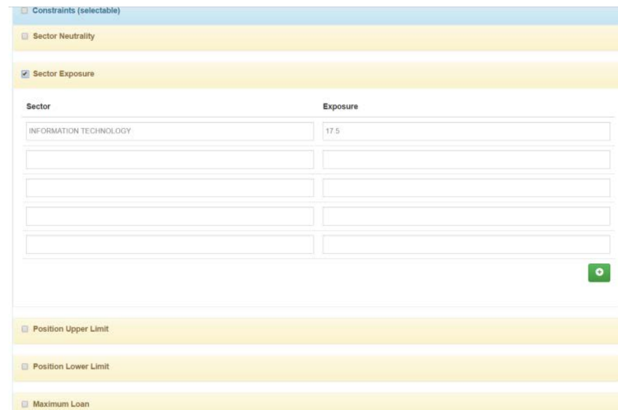
- Many other constraints can be modeled as SOCP constraints.
- NM has a collection of them.

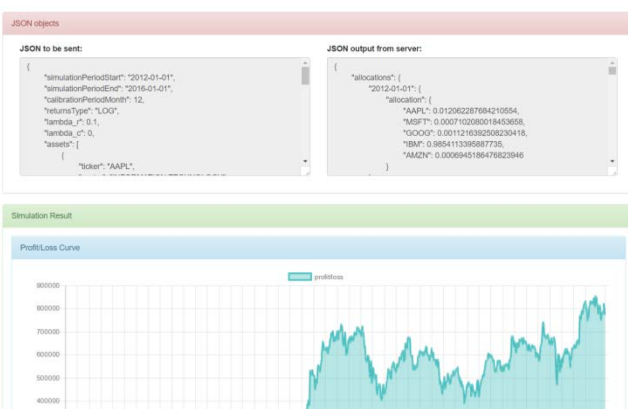


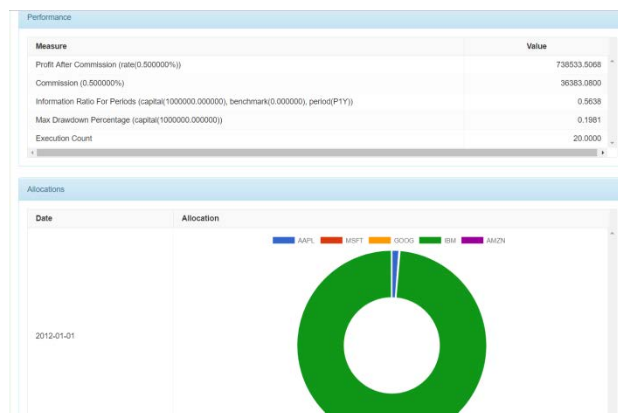
NM SOCP Optimizer

- https://sscloud-201608.appspot.com/socp-portfolio-optim.html



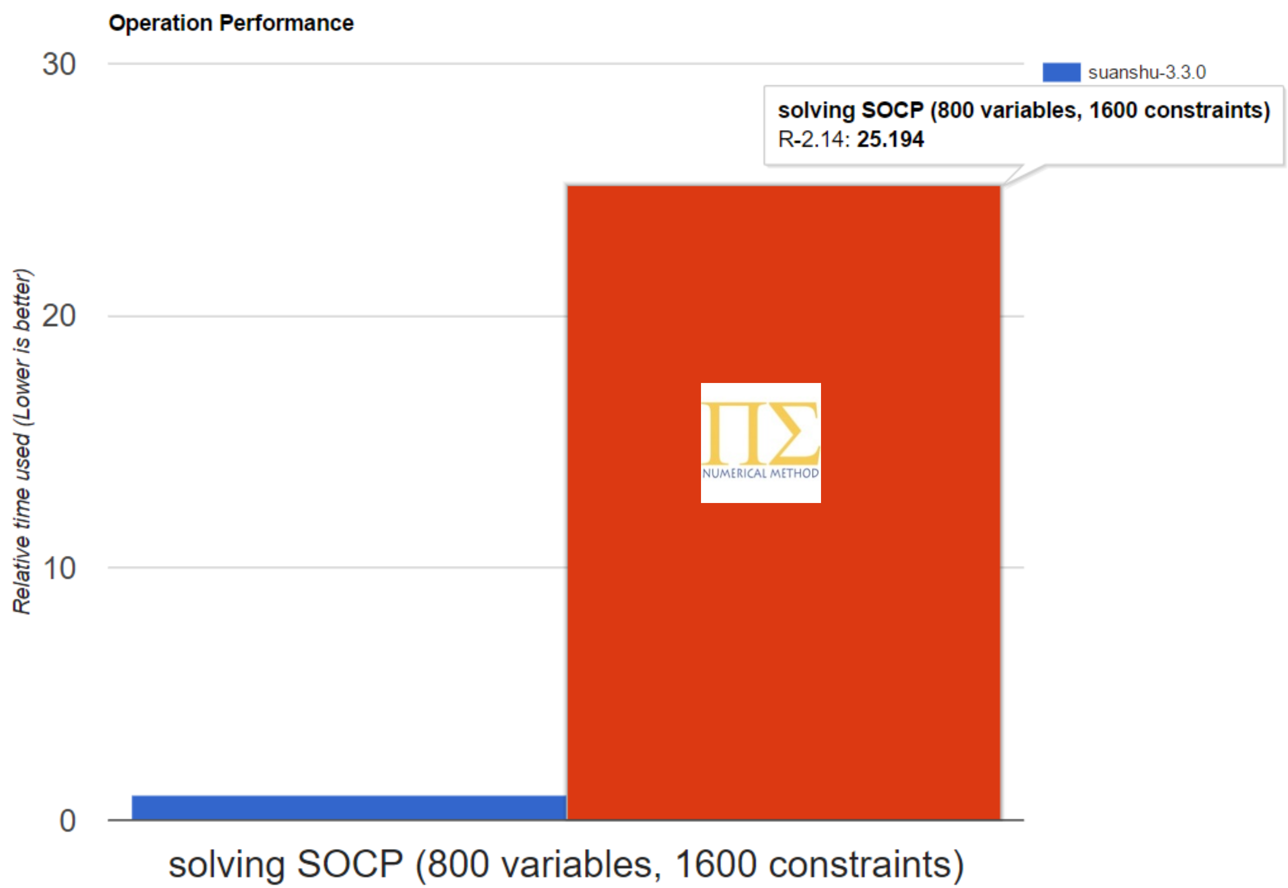






SOCP Optimizers

- Numerical Method Optimizers
  - 25 times faster than free optimizers, e.g., R
- MOSEK
- Gurobi
- CPLEX
- XPRESS



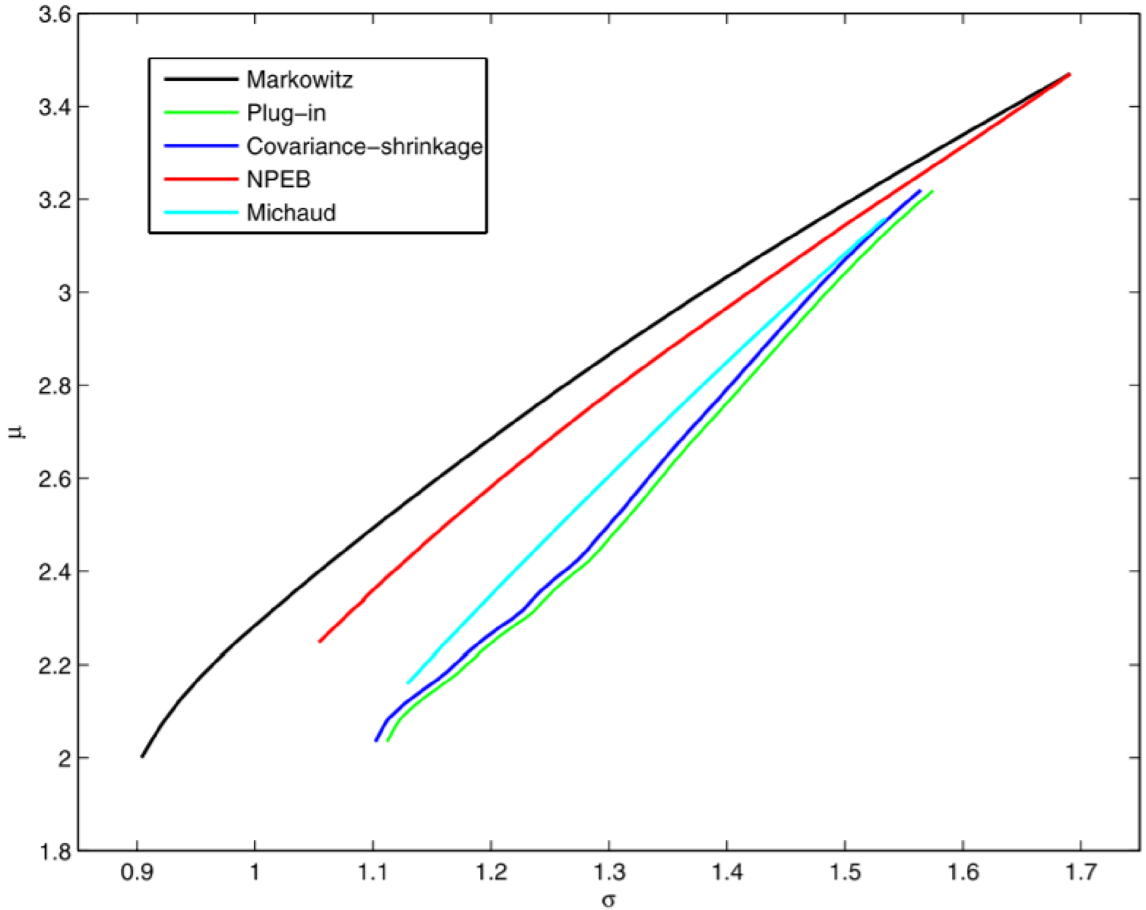
Solution to Performance – Better Estimations

- We combine all the NM modules and algorithms to create better MVO models.
  - Better mean estimation
  - Better covariance estimation
  - Better constraint modeling
  - Better diversification criterion
- NM MVO comparison framework:
  - http://redmine.numericalmethod.com/projects/public/repository/svn-algoquant/show/core/src/main/java/com/numericalmethod/algoquant/model/portfoliooptimization/simulation

Solution to Performance – Unknown Mean and Unknown Covariance

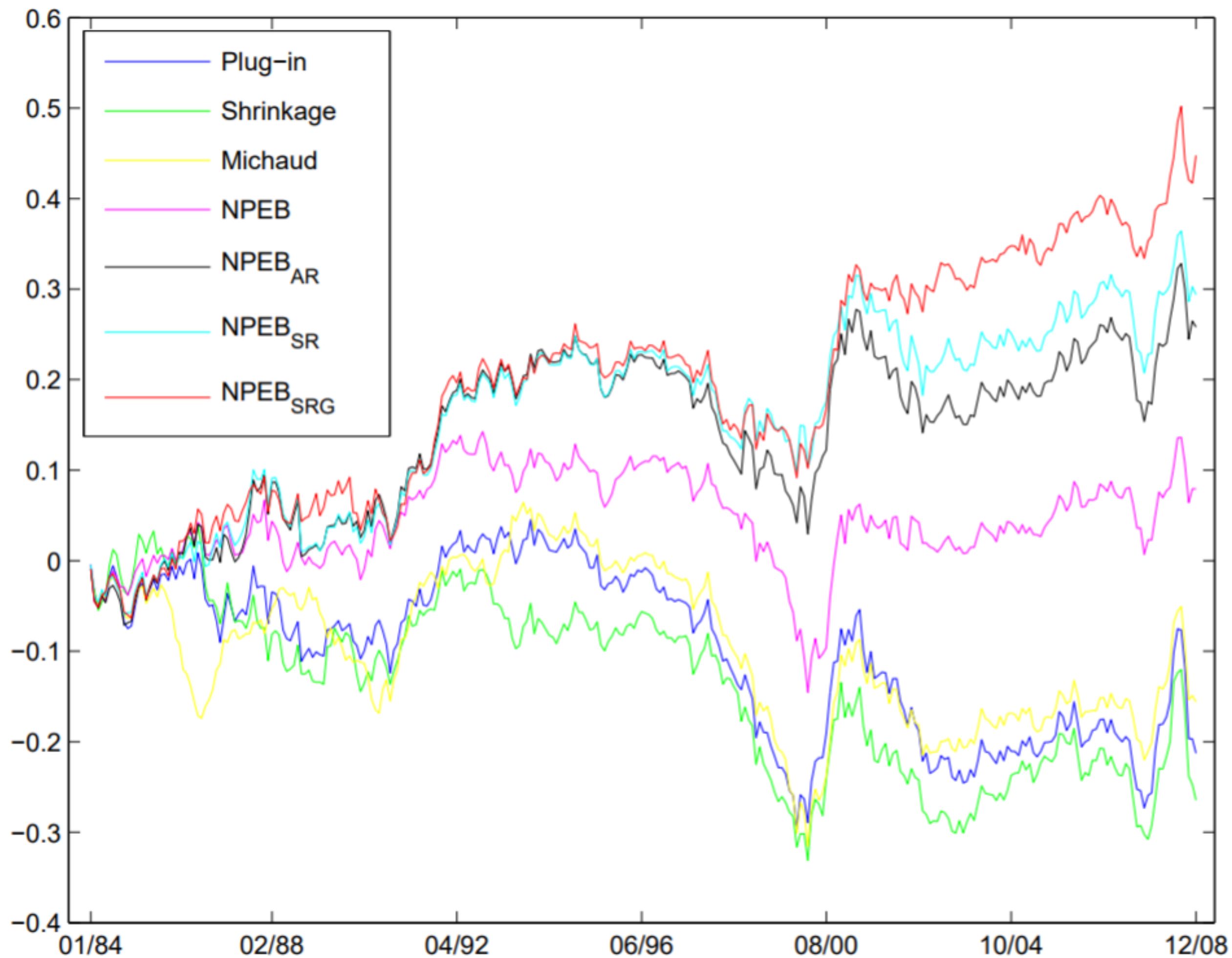
- Incorporate uncertainties of estimations into the model.
  - $\max_{\omega}\{\mathrm{E}(\omega' r_{t+1}) - \lambda\, \mathrm{Var}(\omega' r_{t+1})\}$
- This is a stochastic optimization problem.
  - Use bootstrapping to estimate μ_n and V_n from past return.
    - Resample with replacements
    - Model returns as AR
    - Model returns as SR
    - Model returns as SR+GARCH
- NM:
  - http://numericalmethod.com/blog/2013/02/16/mean-variance-portfolio-optimization-when-means-and-covariances-are-unknown/
  - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/model/lai2010/package-summary.html
  - http://redmine.numericalmethod.com/projects/public/repository/svn-algoquant/show/core/src/main/java/com/numericalmethod/algoquant/model/portfoliooptimization/lai2010

Solution to Performance – Unknown Mean and Unknown Covariance



(σ, μ) *curves of different portfolios.*

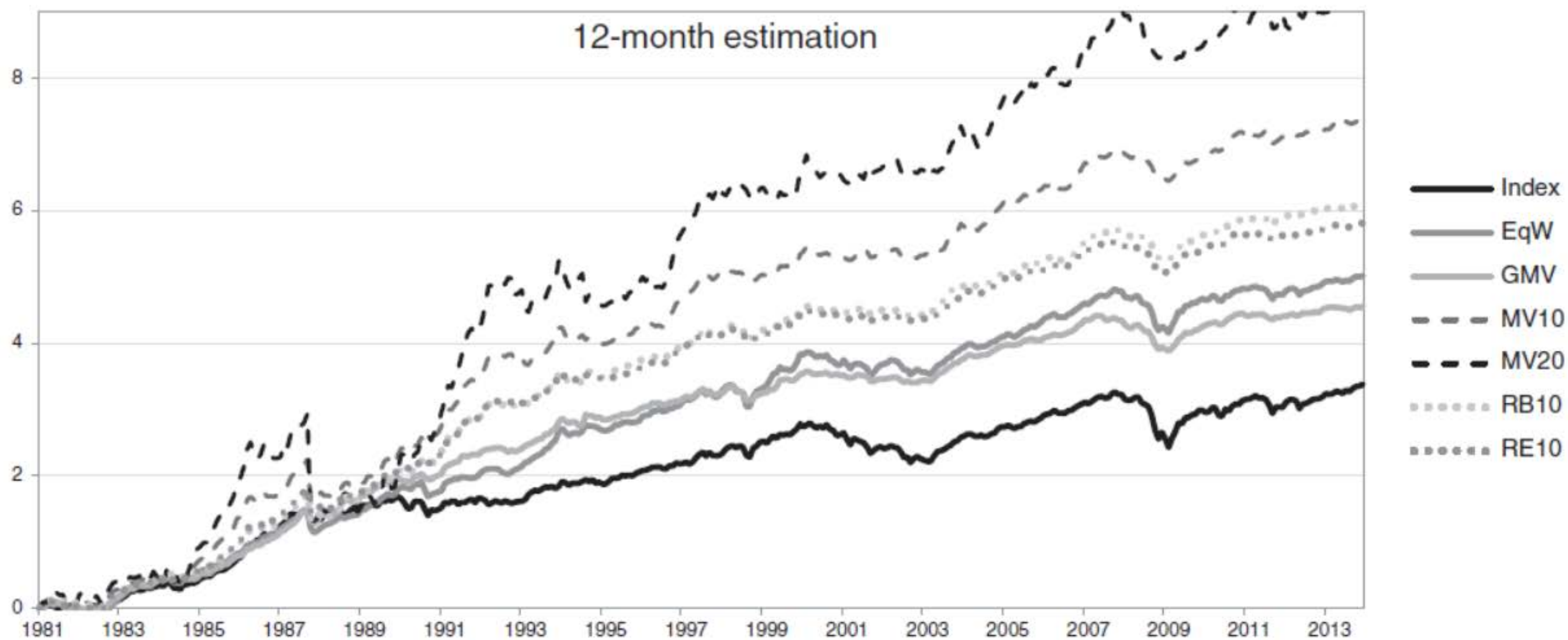
Realized Cumulative Returns Over Time – Unknown Mean and Unknown Covariance



Robust Optimization – Estimation Errors With Bounds

- We assume that there are inherent uncertainties in the inputs, mean and covariance. While the true values of the model's parameters are not known with certainty, but the bounds are assumed to be known.
- The optimal solution represents the best choice when considering all possibilities from the uncertainty set.
- Robust formulation with uncertainty in expected returns.
  - $\min_{\omega} \max_{\hat{\mu} \in U} \{\omega' \Sigma \omega - \lambda \hat{\mu}' \omega\}$
  - It says: minimize the worst of risk among all possible values of the expected return.
- Robust formulation of the MVO problem.
  - $\max_{\omega} \min_{(\hat{\mu},\hat{\Sigma}) \in U_{(\hat{\mu},\hat{\Sigma})}} \{\hat{\mu}' \omega - \lambda \omega' \hat{\Sigma} \omega\}$
  - It says: maximize the worst of risk-adjusted reward among all possible values of mean and covariance.

Robust Optimization – Performance



Multi-Stage Portfolio Optimization

- We can rebalance the portfolio periodically at times $t = 1, \dots, T - 1$.
- Our objective function should be with respect to the expiry time, T.
  - $\max E[U(W_T)]$
- At stage $t = 1$, we can rebalance the portfolio by specifying the weights.
- At stage $t = 2$, we know the realized returns in the last period so we can use this information to rebalance the portfolio. Thus, the weights in stage 2 are functions of the (random) realization in the last stage.
- Solution: stochastic programming, dynamic programming

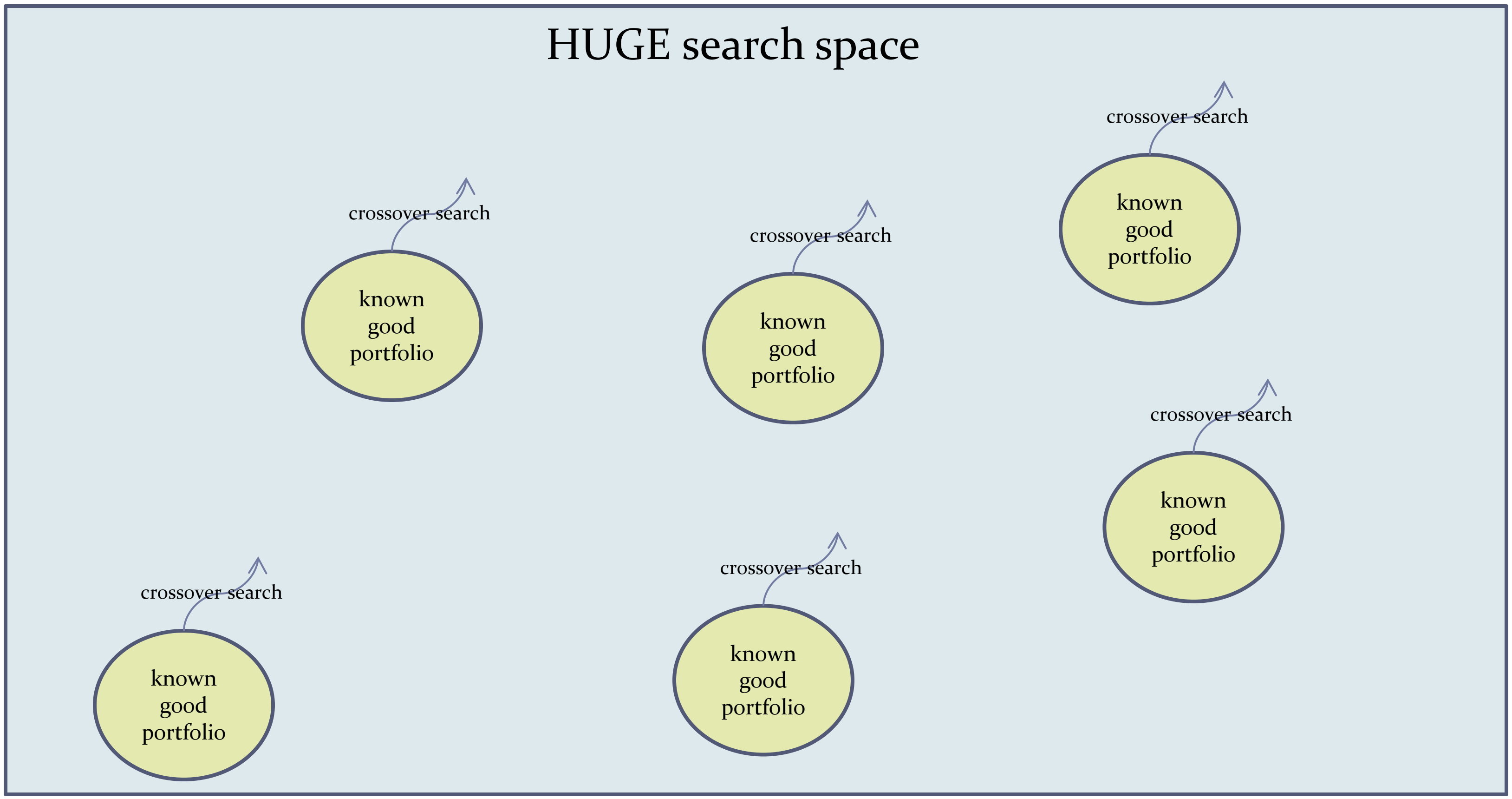
AI – Genetic Programming (1)

- Non-parametric, non-analytical, no estimation algorithm
- Grid search, no math needed
  - But impractical for large number of stocks
  - E.g., 10 levels, 3000 stocks, search space = 10^3000

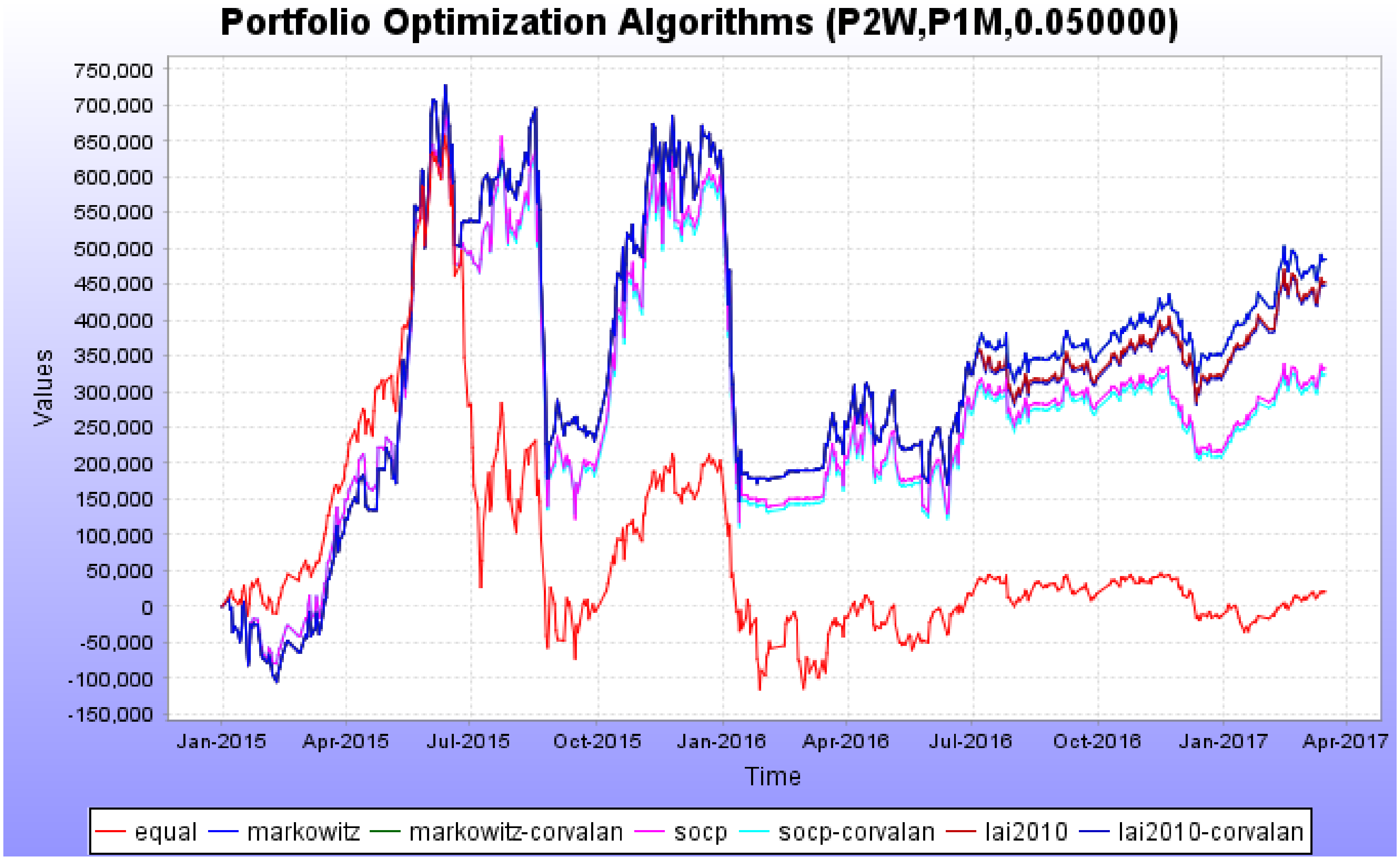
w_{SPY}	w_{TLT}	return r	volat v
0%	**100%**	-0.7%	8.8%
10%	**90%**	4.5%	7.6%
20%	**80%**	9.7%	6.7%
30%	**70%**	14.9%	6.0%
40%	**60%**	20.0%	5.7%
50%	**50%**	25.1%	5.9%
60%	**40%**	30.3%	6.5%
70%	**30%**	35.3%	7.5%
80%	**20%**	40.4%	8.6%
90%	**10%**	45.5%	9.9%
100%	**0%**	50.5%	11.3%

AI – Genetic Programming (2)

- Use AI to improve performance of known portfolios.



Comparisons of Optimization Algorithms



Sharpe-Omega, a Better Measure of Risk

Better Risk Measures

- Variance, hence Sharpe ratio, is not a good measure of risks.
  - Sharpe ratio does not differentiate between winning and losing trades, essentially ignoring their likelihoods (odds).
  - Sharpe ratio does not consider, essentially ignoring, all higher moments of a return distribution except the first two, the mean and variance.
- Other risk measures:
  - Sortino ratio, $S = \frac{R - T}{DR}$
  - Calmar ratio, $C = \frac{\bar{r}_{36}}{MD}$

Sharpe's Choice

- Both A and B have the same mean.
- A has a smaller variance.
- Sharpe will always chooses a portfolio of the smallest variance among all those having the same mean.
  - Hence A is preferred to B by Sharpe.

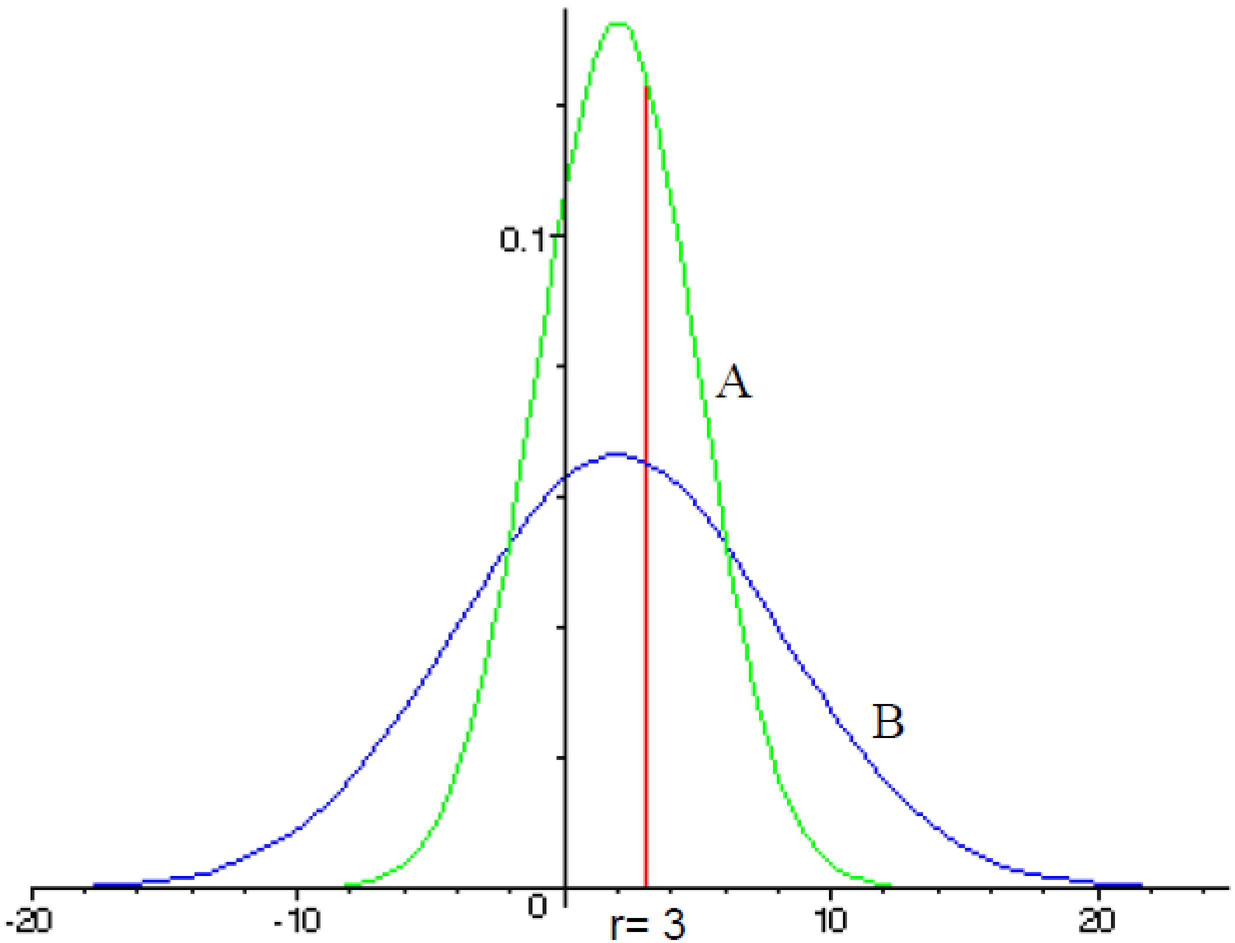
Avoid Downsides and Upsides

- Sharpe chooses the smallest variance portfolio to reduce the chance of having extreme losses.
- Yet, for a Normally distributed return, the extreme gains are as likely as the extreme losses.
- Ignoring the downsides will inevitably ignore the potential for upsides as well.

Potential for Gains

- Suppose we rank A and B by their potential for gains, we would choose B over A.
- Shall we choose the portfolio with the biggest variance then?
  - It is very counter intuitive.

Example 1: A or B?



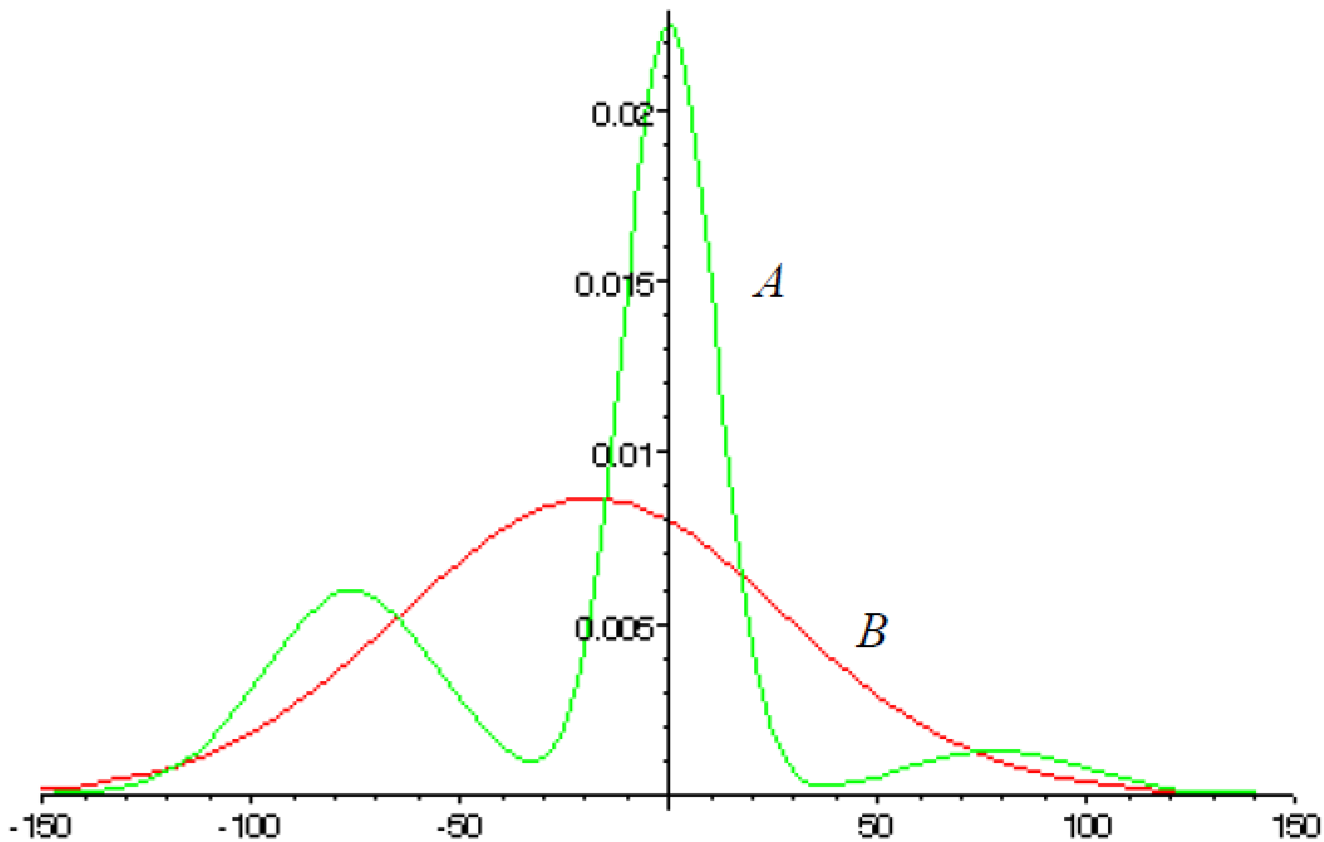
Example 1: L = 3

- Suppose the loss threshold is 3.
- Pictorially, we see that B has more mass to the right of 3 than that of A.
  - B: 43% of mass; A: 37%.
- We compare the likelihood of winning to losing.
  - B: 0.77; A: 0.59.
- We therefore prefer B to A.

Example 1: L = 1

- Suppose the loss threshold is 1.
- A has more mass to the right of L than that of B.
- We compare the likelihood of winning to losing.
  - A: 1.71; B: 1.31.
- We therefore prefer A to B.

Example 2



Example 2: Winning Ratio

- It is evident from the example(s) that, when choosing a portfolio, the likelihoods/odds/chances/potentials for upside and downside are important.
- Winning ratio $\frac{W_A}{W_B}$:
  - 2σ gain: 1.8
  - 3σ gain: 0.85
  - 4σ gain: 35

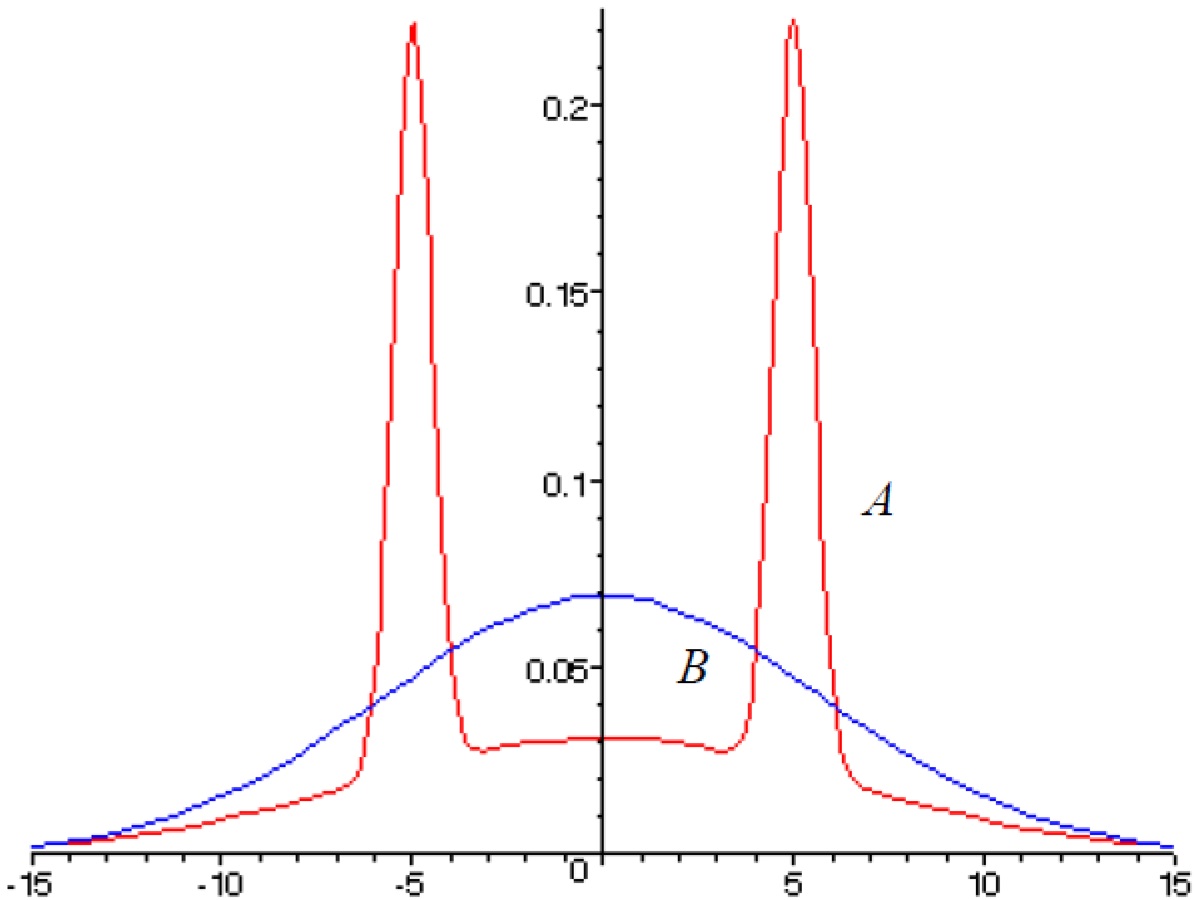
Example 2: Losing Ratio

- Losing ratio $\frac{L_A}{L_B}$:
  - 1σ loss: 1.4
  - 2σ loss: 0.7
  - 3σ loss : 80
  - 4σ loss : 100,000!!!

Higher Moments Are Important

- Both large gains and losses in example 2 are produced by moments of order 5 and higher.
  - They even shadow the effects of skew and kurtosis.
  - Example 2 has the same mean and variance for both distributions.
- Because Sharpe Ratio ignores all moments from order 3 and bigger, it treats all these very different distributions the same.

How Many Moments Are Needed?



Distribution A

- Combining 3 Normal distributions
  - N(-5, 0.5)
  - N(0, 6.5)
  - N(5, 0.5)
- Weights:
  - 25%
  - 50%
  - 25%

Moments of A

- Same mean and variance as distribution B.
- Symmetric distribution implies all odd moments (3^{rd}, 5^{th}, etc.) are 0.
- Kurtosis = 2.65 (smaller than the 3 of Normal)
  - Does smaller Kurtosis imply smaller risk?
- 6^{th} moment: 0.2% different from Normal
- *8^{th} moment: 24% different from Normal*
- *10^{th} moment: 55% bigger than Normal*

Performance Measure Requirements

- Take into account the odds of winning and losing.
- Take into account the sizes of winning and losing.
- Take into account of (all) the moments of a return distribution.

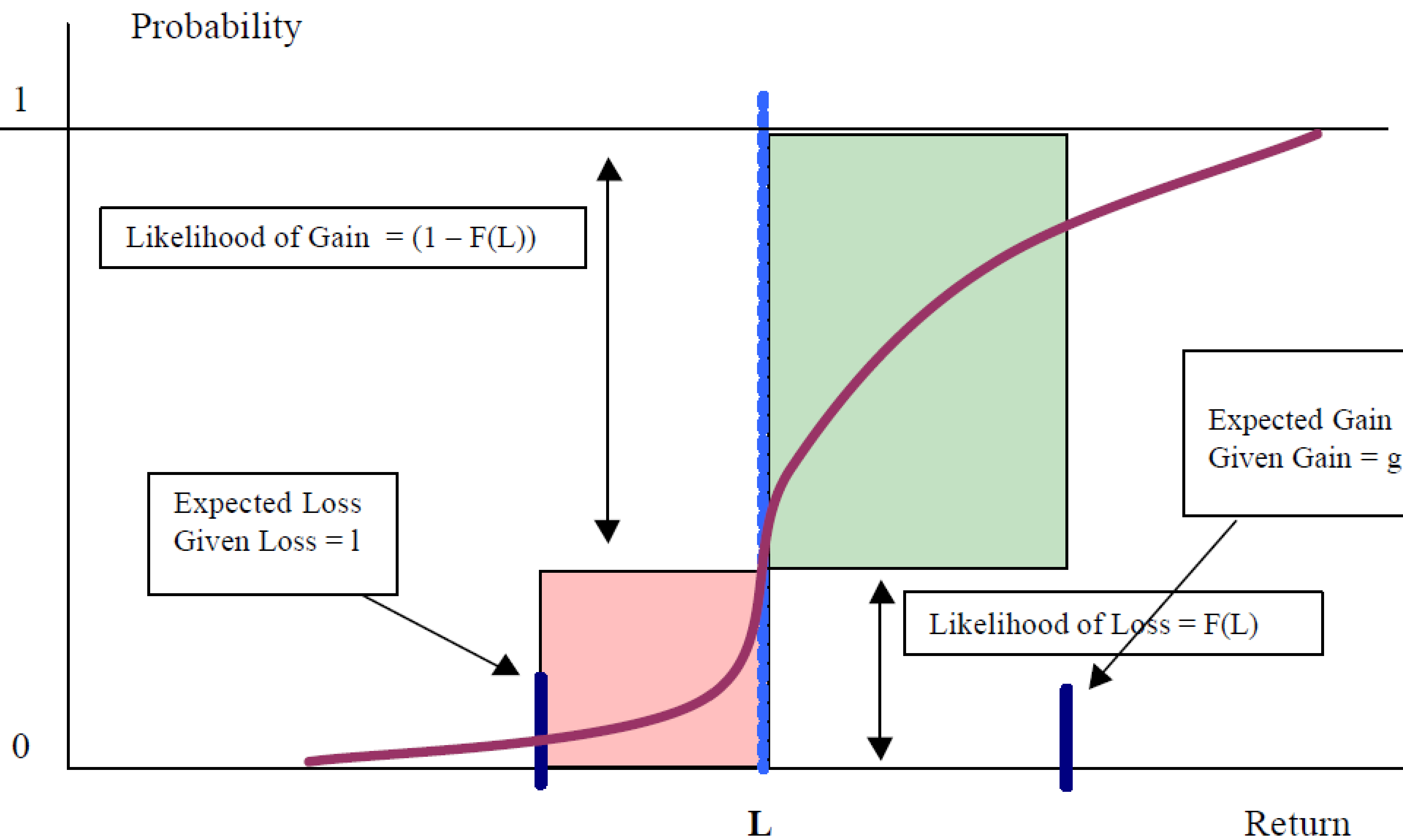
Loss Threshold

- Clearly, the definition, hence likelihoods, of winning and losing depends on how we define loss.
- Suppose L = Loss Threshold,
  - for return < L, we consider it a loss
  - for return > L, we consider it a gain

An Attempt

- To account for
  - the odds of wining and losing
  - the sizes of wining and losing
- We consider
  - $\Omega = \frac{E(r|r>L)\times P(r>L)}{E(r|r\leq L)\times P(r\leq L)}$
- $\Omega = \frac{E(r|r>L)\left(1-F(L)\right)}{E(r|r\leq L)F(L)}$

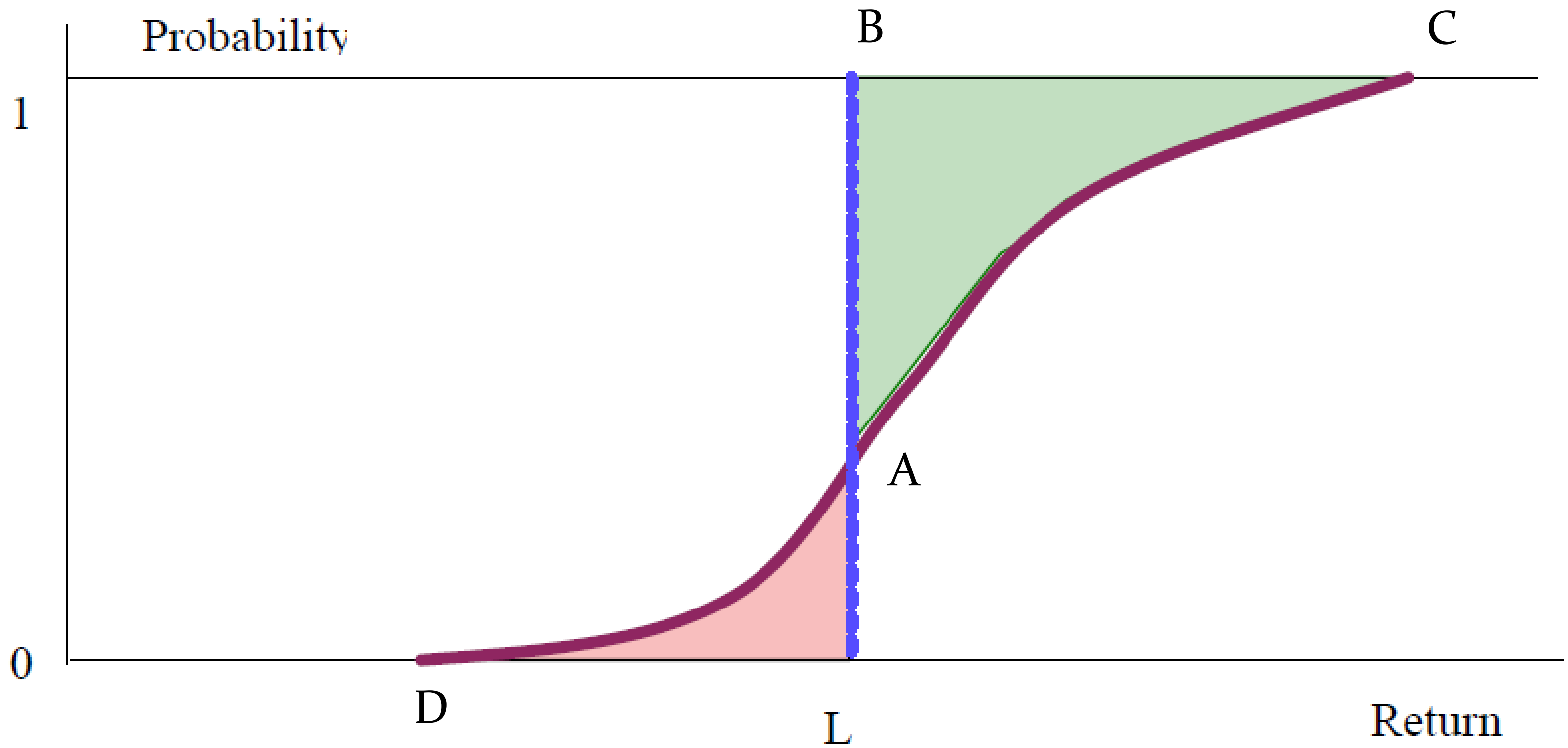
First Attempt



First Attempt Inadequacy

- Why F(L)?
- Not using the information from the entire distribution.
  - hence ignoring higher moments

Another Attempt



Omega Definition

- Ω takes the concept to the limit.
- Ω uses the whole distribution.
- Ω definition:
  - $\Omega = \frac{ABC}{ALD}$
- $\Omega = \frac{\int_{L}^{b=\max\{r\}}[1-F(r)]dr}{\int_{a=\min\{r\}}^{L} F(r)dr}$

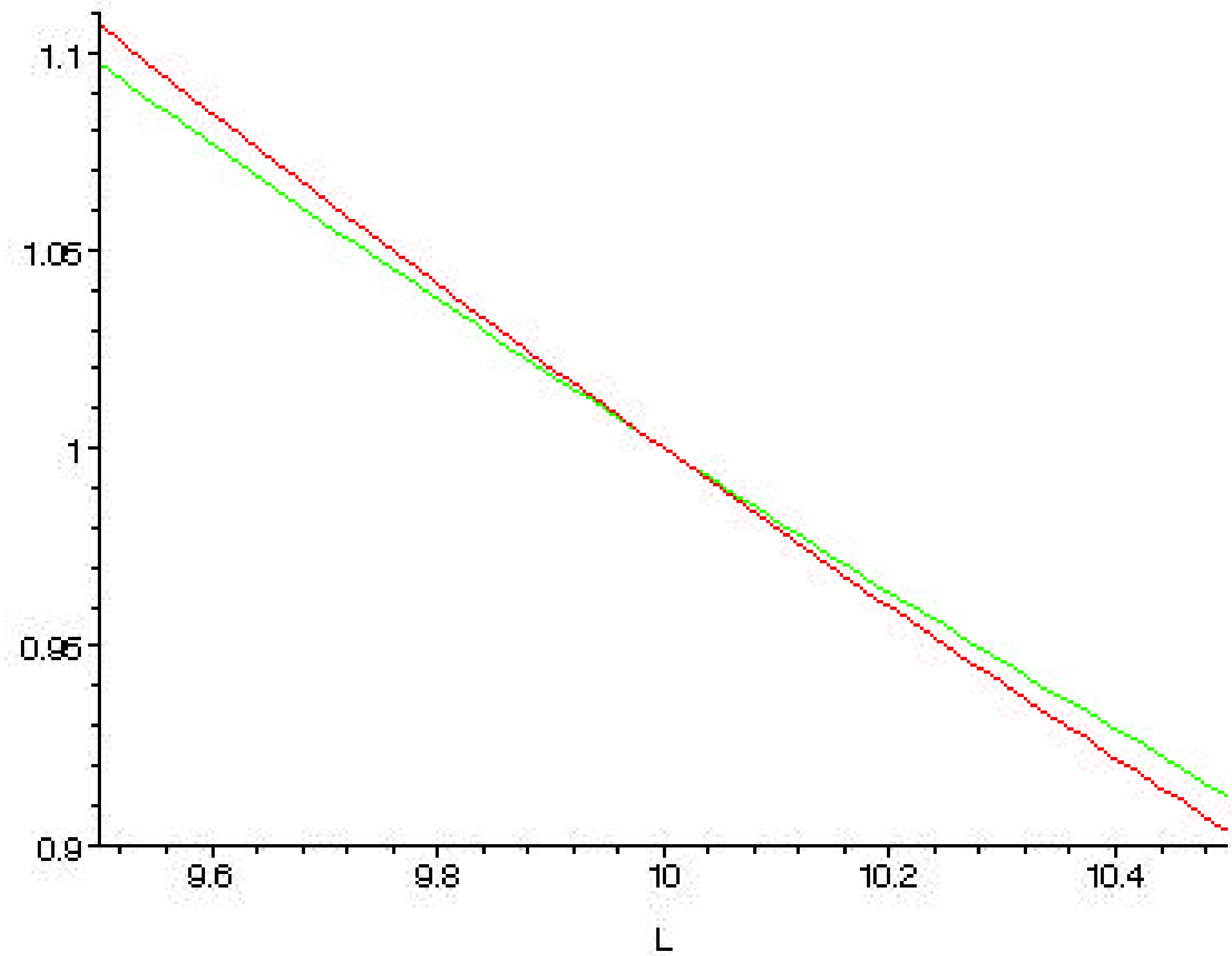
Intuitions

- Omega is a ratio of winning size weighted by probabilities to losing size weighted by probabilities.
- Omega considers size and odds of winning and losing trades.
- Omega considers all moments because the definition incorporates the whole distribution.

Omega Advantages

- There is no parameter (estimation).
- There is no need to estimate (higher) moments.
- Work with all kinds of distributions.
- Use a function (of Loss Threshold) to measure performance rather than a single number (as in Sharpe Ratio).
- It is as smooth as the return distribution.
- It is monotonic decreasing.

Omega Example



Numerator Integral (1)

- $\int_L^b d\big[x\big(1 - F(x)\big)\big]$
- $= \big[x\big(1 - F(x)\big)\big]^b{}_L$
- $= b\big(1 - F(b)\big) - L\big(1 - F(L)\big)$
- $= -L\big(1 - F(L)\big)$

Numerator Integral (2)

- $\int_L^b d\left[x\left(1 - F(x)\right)\right]$
- $= \int_L^b \left(1 - F(x)\right) dx + \int_L^b x d\left(1 - F(x)\right)$
- $= \int_L^b \left(1 - F(x)\right) dx - \int_L^b x dF(x)$

Numerator Integral (3)

- $-L\big(1 - F(L)\big) = \int_L^b \big(1 - F(x)\big)dx - \int_L^b xdF(x)$
- $\int_L^b \big(1 - F(x)\big)dx = -L\big(1 - F(L)\big) + \int_L^b xdF(x)$
- $= \int_L^b (x - L)f(x)dx$
- $= \int_a^b \max(x - L, 0)f(x)dx$
- $= E[\max(x - L, 0)]$

undiscounted call option price

Denominator Integral (1)

- $\int_a^L d[xF(x)]$
- $= [xF(x)]^L{}_a$
- $= LF(L) - a\big(F(a)\big)$
- $= LF(L)$

Denominator Integral (2)

- $\int_a^L d[xF(x)]$
- $= \int_a^L F(x)dx + \int_a^L xdF(x)$

Denominator Integral (3)

- $LF(L) = \int_a^L F(x)dx + \int_a^L x dF(x)$
- $\int_a^L F(x)dx = LF(L) - \int_a^L x dF(x)$
- $= \int_a^L (L - x) f(x) dx$
- $= \int_a^b \max(L - x, 0) f(x) dx$
- $= E[\max(L - x, 0)]$

undiscounted put option price

Another Look at Omega

- $\Omega = \frac{\int_L^{b=\max\{r\}}[1-F(r)]dr}{\int_{a=\min\{r\}}^L F(r)dr}$
- $= \frac{E[\max(x-L,0)]}{E[\max(L-x,0)]}$
- $= \frac{e^{-r_f}E[\max(x-L,0)]}{e^{-r_f}E[\max(L-x,0)]}$
- $= \frac{C(L)}{P(L)}$

Options Intuition

- Numerator: the cost of acquiring the return above L
- Denominator: the cost of protecting the return below L
- Risk measure: the put option price as the cost of protection is a much more general measure than variance

Can We Do Better?

- Excess return in Sharpe Ratio is more intuitive than $C(L)$ in Omega.
- Put options price as a risk measure in Omega is better than variance in Sharpe Ratio.

Sharpe-Omega

- $\Omega_S = \frac{\bar{r} - L}{P(L)}$
- In this definition, we combine the advantages in both Sharpe Ratio and Omega.
  - meaning of excess return is clear
  - risk is bettered measured
- Sharpe-Omega is more intuitive.
- *Ω_S ranks the portfolios in exactly the same way as Ω.*

Sharpe-Omega and Moments

- It is important to note that the numerator relates only to the first moment (the mean) of the returns distribution.
- It is the denominator that take into account the variance and all the higher moments, hence the whole distribution.

Sharpe-Omega and Variance

- Suppose $\bar{r} > L$. $\Omega_S > 0$.
  - The bigger the volatility, the higher the put price, the bigger the risk, the smaller the Ω_S, the less attractive the investment.
  - We want smaller volatility to be more certain about the gains.
- Suppose $\bar{r} < L$. $\Omega_S < 0$.
  - The bigger the volatility, the higher the put price, the bigger the Ω_S, the more attractive the investment.
  - Bigger volatility increases the odd of earning a return above L.

Non-Linear, Non-Convex Portfolio Optimization

- In general, a Sharpe optimized portfolio is different from an Omega optimized portfolio.

Beyond Mean Variance Optimization

Optimizing for Omega

- $$\begin{cases} \max_{x} \Omega_S(x) \\ \sum_i^n x_i E(r_i) \geq \rho \\ \sum_i^n x_i = 1 \\ {x_i}^l \leq x_i \leq 1 \end{cases}$$
- Minimum holding: $x^l = \left({x_1}^l, \ldots, {x_n}^l\right)'$

Optimization Methods

- Nonlinear Programming
  - Penalty Method
- Global Optimization
  - Tabu search (Glover 2005)
  - Threshold Accepting algorithm (Avouyi-Dovi et al.)
  - MCS algorithm (Huyer and Neumaier 1999)
  - Simulated Annealing
  - Genetic Algorithm
- Integer Programming (Mausser et al.)

3 Assets Example

- $x_1 + x_2 + x_3 = 1$
- $R_i = x_1 r_{1i} + x_2 r_{2i} + x_3 r_{3i}$
- $= x_1 r_{1i} + x_2 r_{2i} + (1 - x_1 - x_2) r_{3i}$

Penalty Method

- $F(x_1, x_2) = -\Omega(R_i) + \rho\{[\min(0, x_1)]^2 + [\min(0, x_2)]^2 + [\min(0,1 - x_1 - x_2)]^2\}$
- Can apply Nelder-Mead, a Simplex algorithm that takes initial guesses.
- F needs not be differentiable.
- Can do random-restart to search for global optimum.

Threshold Accepting Algorithm

- It is a local search algorithm.
  - It explores the potential candidates around the current best solution.
- It "escapes" the local minimum by allowing choosing a lower than current best solution.
  - This is in very sharp contrast to a hilling climbing algorithm.

Objective

- Objective function
  - $h: X \rightarrow R, X \in R^n$
- Optimum
  - $h_{\text{opt}} = \max_{x \in X} h(x)$

Initialization

- Initialize n (number of iterations) and $step$.
- Initialize sequence of thresholds th_k, $k = 1, \ldots, step$
- Starting point: $x_0 \in X$

Thresholds

- Simulate a set of portfolios.
- Compute the distances between the portfolios.
- Order the distances from smallest to biggest.
- Choose the first $step$ number of them as thresholds.

Search

- $x_{i+1} \in N_{x_i}$ (neighbour of x_i)
- Threshold: $\Delta h = h(x_{i+1}) - h(x_i)$
- Accepting: If $\Delta h > -th_k$ set $x_{i+1} = x_i$
- Continue until we finish the last (smallest) threshold.
  - $h(x_i) \approx h_{opt}$
- Evaluating h by Monte Carlo simulation.

AI – Genetic Programming

- Those arbitrary, non-convex, non-differentiable, non-continuous, noisy, objective functions are difficult to be optimized using traditional methods. We resort to Artificial Intelligence, heuristics and simulations.
- In a genetic algorithm, a population of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem is evolved toward better solutions. Each candidate solution has a set of properties (its chromosomes or genotype) which can be mutated and altered; traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible.
- NM Genetic Programming Framework:
  - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/optimization/multivariate/geneticalgorithm/package-summary.html

Differential Evolution

- DE is used for multidimensional real-valued functions but does not use the gradient of the problem being optimized, which means DE does not require for the optimization problem to be differentiable as is required by classic optimization methods such as gradient descent and quasi-newton methods. DE can therefore also be used on optimization problems that are not even continuous, are noisy, change over time, etc.
- DE optimizes a problem by maintaining a population of candidate solutions and creating new candidate solutions by combining existing ones according to its simple formulae, and then keeping whichever candidate solution has the best score or fitness on the optimization problem at hand. In this way the optimization problem is treated as a black box that merely provides a measure of quality given a candidate solution and the gradient is therefore not needed.
- NM:
  - http://numericalmethod.com/blog/2011/05/31/strategy-optimization/
  - http://www.numericalmethod.com/javadoc/suanshu/com/numericalmethod/suanshu/optimization/multivariate/geneticalgorithm/minimizer/deoptim/DEOptim.html

Multi Factor Model

Fundamental Theorem in Quantitative Trading

- The average return of a stock = payoff for taking risk
  - = factor exposure * factor premium
- Factor exposure: the exposure of a stock to some kind of risk (or factor)
- Factor premium: the payoff to an investor per one unit of exposure

Fundamental Factor Model

- Fundamental factors = stock characteristics.
  - size, P/E, current ratio, advertising-expenditure-to-sales ratio, analyst rating, M12M,
- Factor exposure is known.
  - The exposure to the risk or factor "size" is simply size/capitalization.
  - The exposure to the risk or factor "P/E" is simply P/E.
- Factor premium needs to be estimated.
  - The premium or payoff to one unit of exposure to size is unknown.
- $r_i = \alpha_i + \beta_{i1} f_1 + \beta_{i2} f_2 + \cdots + \beta_{iK} f_K + \epsilon_i$
  - K: number of factors
  - β_{ij}: the exposure of each stock i to the j-th factor is different
  - f_j: the factor premium is a property of the factor and is independent of stocks
  - α_i: time invariant individual stock effect
  - The uncertainty of r_i comes from the uncertainty of f_j, which are themselves random variable.

Economic Factor Models

- Economic factors: factor premiums/effects same for all stocks, e.g., inflation, but different stocks have different exposures to them.
- Factor exposures need to be estimated: how much is a stock exposed (sensitive/affected by) to inflation?
- Assumption: the unknown true premium of a factor is a linear combination of the observed factor value and a constant (which takes care of the expected part of the factor value).
  - We are only rewarded for the unexpected part of the factor (value).

Quintiles Method

- To test whether a factor (or a strategy) is significant in generating alpha…
- Rank/sort the stocks in a universe by the factor by standardized factor exposures, e.g., z-scores.
  - $z_i = \frac{\beta_i - \mu}{\sigma}$
- Divide them into 5 groups (20% each).
- Portfolio formed each quarter over the test period. Each portfolio is hold for 12 months.
  - Number of portfolios in each quintile for the test period = test period (in years)*4*(size of universe/5).
- Compute average returns for each quintile.
- Factor significant if
  - top first quintile significantly outperformed the universe
  - the bottom fifth quintile significantly underperformed
  - the outperformance/underperformance was consistent over time

Economic Factor Models – Math

- $r_i = \alpha_i + \beta_{i1} f_1 + \beta_{i2} f_2 + \cdots + \beta_{iK} f_K + \epsilon_i$
  - $= [\alpha_i, \quad \beta_{i1}, \quad \dots, \quad \beta_{iK}] \begin{bmatrix} 1 \\ f_1 \\ \dots \\ f_K \end{bmatrix} + \epsilon_i$
  - $= \boldsymbol{\beta_i}' \boldsymbol{f} + \epsilon_i$
- $\mathrm{E}(r_i) = \boldsymbol{\beta_i}' \, \mathrm{E}(\boldsymbol{f})$

Variance Risk

- $r_i = \boldsymbol{\beta_i}'\boldsymbol{f} + \epsilon_i$
- Total risk = diversifiable risk + non-diversifiable risk
  - $\mathrm{Var}(r_i) = \mathrm{Var}\left(\boldsymbol{\beta_i}'\boldsymbol{f}\right) + \mathrm{Var}(\epsilon_i)$
  - $= \boldsymbol{\beta_i}'\,\mathrm{Var}(\boldsymbol{f})\,\boldsymbol{\beta_i} + \mathrm{Var}(\epsilon_i)$
  - $\mathrm{Var}(\boldsymbol{f})$ is the variance-covariance matrix of the factor premiums.

Economic Factor Model – Factor Premiums

- Economic/Behavior/Market: usually expressed as rates, e.g., change %
- Fundamental/Technical/Analyst: zero investment portfolio method
  - For each time t, for each factor k, set the upper and lower cutoff points, $\overline{x_k}$ and $\underline{x_k}$.
  - Divide the stocks into three groups.
    - High group: $x_{ikt} > \overline{x_k}$
    - Low group: $x_{ikt} < \overline{x_k}$
    - Others
  - Factor premium is the expected return to the zero-investment position that put \$1 into the high group and short \$1 in the low group.
  - $f_{kt} = \mathrm{E}(r_t\ |x_{kt} > \overline{x_k}) - \mathrm{E}(r_t|x_{kt} < \overline{x_k})$
    - The expectation is taken across stocks.
    - The weights and returns are both decided on time t.
- Statistical factors: PCA on returns
  - Each of the most significant factor premium is a linear combination of the stock returns at time t.
  - Pick the K eigenvectors q_1, …, q_K that correspond to the largest K eigenvalues.
  - Statistical factors are: $f_{i,t} = q_i{}'r_t$.
- Note: all of them, by construction, change over time.

Economic Factor Model – Factor Exposures

- For each of the *N* stocks, run an OLS to compute the factor exposures/factor sensitivities/factor loadings.
- Report betas and their standard errors.
- Merger:
  - $\hat{\beta}_{AB} = \frac{s_A}{s_A+s_B}\hat{\beta}_A + \frac{s_B}{s_A+s_B}\hat{\beta}_B$
  - s_A: pre-merger market capitalization of firm A
  - s_B: pre-merger market capitalization of firm B
- IPO by Characteristic Matching:
  - We use the factor exposures of *M* similar firms.
  - To identify the *M* similar firms,
    - We choose L company characteristics;
    - Compute the z-score of those L characteristics for a group of firms $\{z_i = (z_{i1}, \ldots, z_{iL})\}$ as well as those of the new company, $z = (z_1, \ldots, z_L)$.
    - Set a threshold, ε.
    - The similar firms are those with smaller distances. That is,
      - $\|z - z_i\| < \varepsilon$.
    - $\hat{\beta} = \frac{1}{M}(\hat{\beta}_1 + \cdots + \hat{\beta}_M)$

Economic Factor Model – Algorithm

1. Set the time interval, e.g. monthly or rebalacing period, and time period of data, e.g. 3 to 5 years.
2. Set the investment universe.
3. Choose the factors for the model.
4. Set the risk-free rate.
5. Collect stock returns for the time period at each interval. If benchmarking,
   1. Better use risk-free rate adjusted return: $r_{it}{}^{*} = r_{it} - r_{ft}$
   2. If for benchmarking, we use residual stock returns in lieu of stock returns.
   3. $\tilde{r}_i \equiv \tilde{\alpha}_i + \tilde{\epsilon}_i = r_i - \tilde{\beta}_i r_B$
6. Collect factor premium data for the time period at each interval.
   1. Economic/behavior/market factors: readily available
   2. Fundamental/technical/analyst factors: zero investment portfolio method
   3. Statistical factors: PCA
7. Estimate the factor exposures from time series regression of stock returns on premium.
   1. If not enough factor-premium data, do characteristic matching before regression.
8. Check robustness by splitting the data into subsets and compare the estimates for each subset. Highlight major differences.
   1. Split by time periods.
   2. Split by sectors.
9. If the estimation is not robust (subset estimates are not similar), try a different estimation method.
10. Compute the expected stock returns.
11. Compute the risks of the stocks for both non-diversifiable and diversifiable risks.
12. Compute the correlation matrix of stocks returns.

NM Technologies

SaaS Tools for Modeling and Constructing "Alpha Strategy"

- Mutual fund and private equity fund managers, quantitative investment teams, etc.
- In China, the majority of the quantitative trading strategies are alpha strategies. They recently have poor performance. Funds are extremely desperate to look for a new direction
- NM's research library provides two tools that can immediately improve the performance of existing alpha strategies
    - multi-factor model
    - asset allocation, portfolio optimization
- Provide users with an intuitive and easy to use interface to complete complicated tasks such as financial modeling, strategy optimization, return performance and risk control, all without programming
    - With our system, traders only need to do the first step, factor selection, and leave the rest of the complex process to system automation, hence great efficiency in strategy research and time to production

Factor selection/definition → **Model construction** → **Portfolio optimization** → **Backtesting** → **Reporting**

Factor selection/definition
- α factors
- sorting, grouping
- factor exposures
- factor premiums

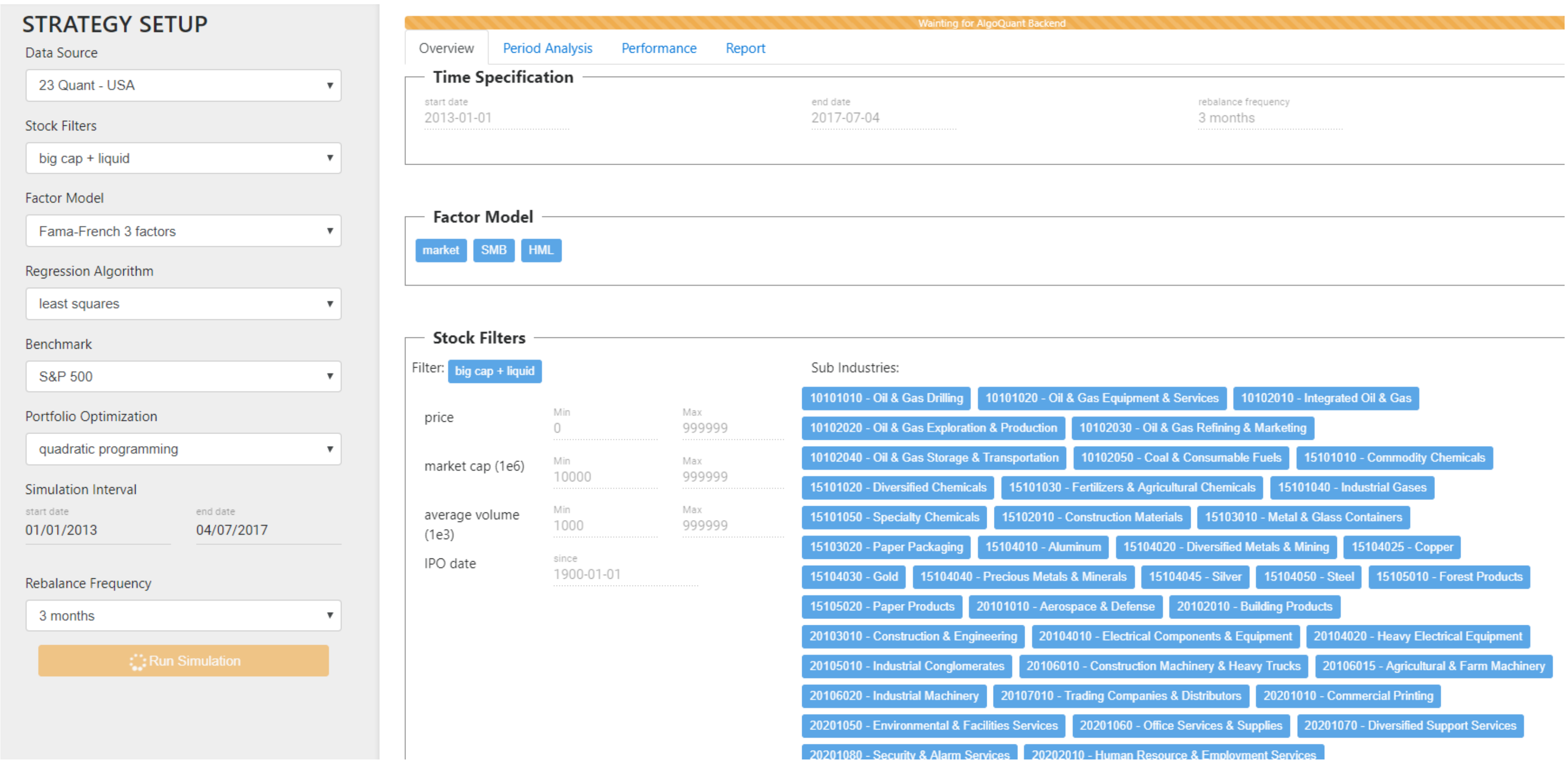
Model construction
- OLS regression
- panel regression
- α、β computation

Portfolio optimization
- Markowitz
- Black-Litterman
- Second Order Conic Programming
- uncertain mean and covariance
- customized objective functions
- corner portfolios

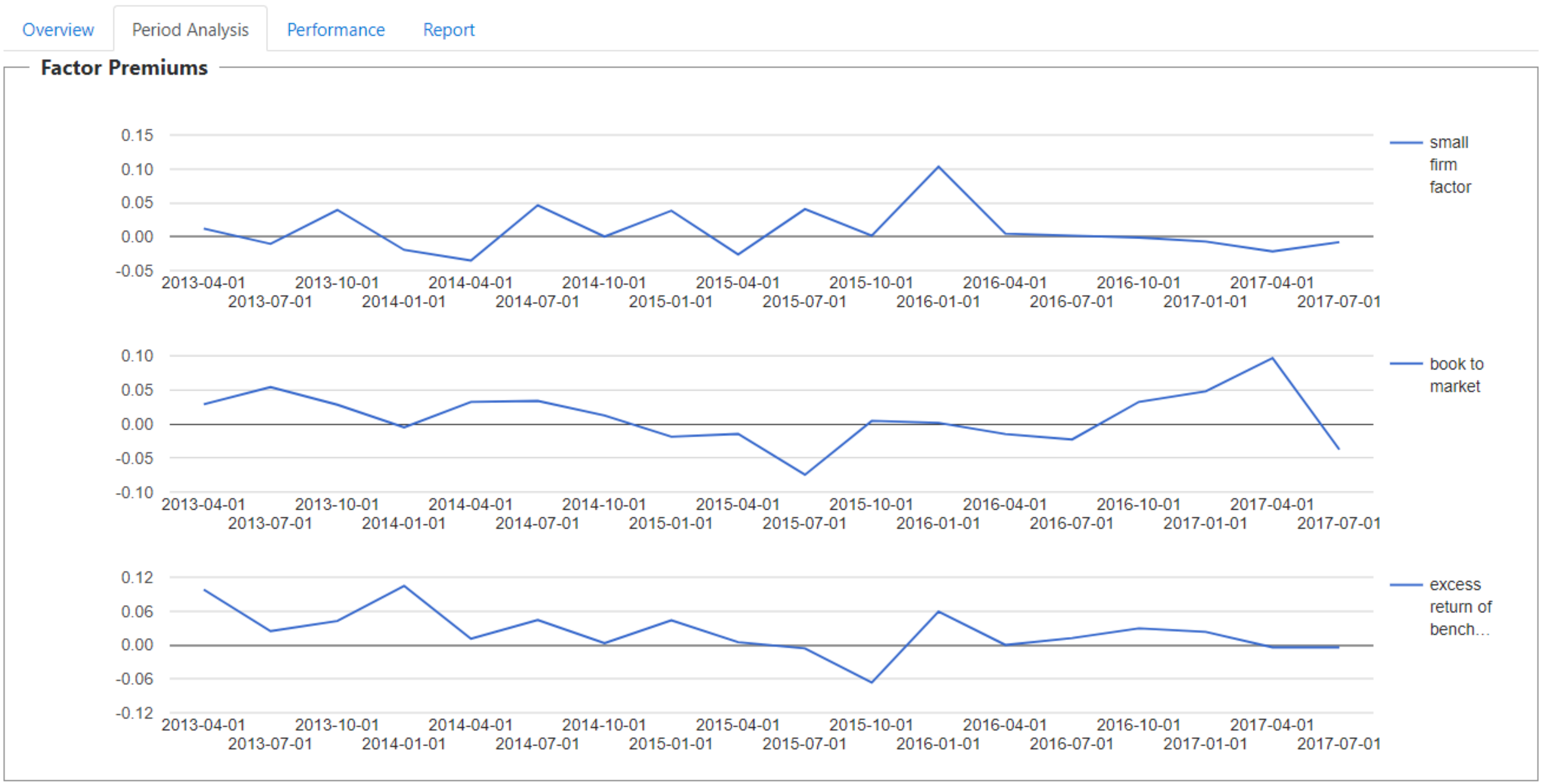
Backtesting
- historical backtesting
- Monte Carlo backtesting
- bootstrapping backtesting
- scenario and stress testing

Reporting
- VaR computation
- p&l attribution
- risk assessment
- easy to understand, professional and standardized report

NM FinTech – Alpha Strategy Framework



NM FinTech – Factor Premiums



NM FinTech – Factor Exposures

Factor Exposures

Date Exposures

2013-04-01

Stock	small firm factor	book to market	excess return of benchmark
PH	-0.9802062405764855	-0.6867595439114671	0.8443270070883196
ECL	0.7521821173681223	0.7712321081549549	0.41600880818156816
ITW	0.17675921120840418	-0.41920848052941834	1.462360491597159
NOV	0.7530674873430164	0.07599742427791317	2.567622437675358
PFG	0.004088871488860885	-0.07467809835274612	-0.07571164710132486
MMM	0.5023827673385409	-0.354506426256311	0.761883422042528
AEP	0.5880941795292267	0.5140433187641414	0.18314623674149513
AES	-0.025190982935465126	-0.009985331505686163	0.02334266228176007
FLR	-0.06415973186078433	-0.34133868542667295	-0.04391923699310265
GE	0.4060312276472955	0.16253436704860358	1.418536085529567

1 2 3 4 5

2013-07-01

Stock	small firm factor	book to market	excess return of benchmark
PH	-0.9912769842028955	-0.8139346415834159	0.7670444876460103
ECL	0.6947912459906902	0.6740344317826733	0.44592739400400006
ITW	0.1353917117792637	-0.7381695235899508	1.3276863607245093
NOV	0.6563596226067306	-0.45258464591312536	2.389047825117468
PFG	0.00406694033146353	-0.05162158855102556	-0.058008607109780515
MMM	0.4924548894933311	-0.2293220456522416	0.8561939022891
AEP	0.6249693540572964	0.6389905663188581	0.20315208474702667
AES	-0.025169263626619424	-0.014320268793186802	0.019933786373609364
FLR	-0.06436154588664873	-0.1291689863470745	0.11898697474930942
GE	0.446109538098187	-0.06433387388928392	1.2126257750693323

1 2 3 4 5

2013-10-01

Stock	small firm factor	book to market	excess return of benchmark
PH	-0.9839797479412057	-0.7677518985321332	0.7732466407304476
ECL	0.7181600495627636	1.0419616614207015	0.46672280760910234
ITW	0.24823659496239897	-0.2429586393855775	1.422667825468856
NOV	0.6036784974107626	-0.5610902184915798	2.34522665669061
PFG	0.09320505750702748	0.5654893921048689	0.03920985389000287
MMM	0.545633759140703	-0.11325601778030671	0.9004568312588244
AEP	0.5069160043371046	0.24502250623654948	0.10431317520553443
AES	-0.02516164488758416	-0.014265813227837012	0.01994343450752461
FLR	-0.06282927101664905	-0.11856091341911142	0.12065814988078041
GE	0.4620571470695346	-0.08703939934079613	1.2256556098033344

1 2 3 4 5

NM FinTech – Stock Selection

Stock Selection

Date | Stocks

2013-04-01

No.	Ticker	Company	Position	Sector	Industry	Country	Market Cap	P/E	Price
1	DUK	company name n/a		UTILITIES	ELECTRIC UTILITIES	United States	45.78B	21.1531879963961	65.01
2	SHW	company name n/a		MATERIALS	CHEMICALS	United States	16.15B	25.7698963391597	156.68
3	PPG	company name n/a		MATERIALS	CHEMICALS	United States	21.27B	22.4479697676161	138.55

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2013-07-01

No.	Ticker	Company	Position	Sector	Industry	Country	Market Cap	P/E	Price
1	DUK	company name n/a		UTILITIES	ELECTRIC UTILITIES	United States	51.02B	21.6019293930254	72.32
2	MPC	company name n/a		ENERGY	OIL GAS AND CONSUMABLE FUELS	United States	29.72B	8.59831946397264	89.83
3	PSX	company name n/a		ENERGY	OIL GAS AND CONSUMABLE FUELS	United States	43.01B	8.83755895611501	69.36

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2013-10-01

No.	Ticker	Company	Position	Sector	Industry	Country	Market Cap	P/E	Price
1	REGN	company name n/a		HEALTH CARE	BIOTECHNOLOGY	United States	22.07B	26.5342890947459	230.14
2	CELG	company name n/a		HEALTH CARE	BIOTECHNOLOGY	United States	49.60B	32.2728678880911	118.91
3	YHOO	company name n/a		INFORMATION TECHNOLOGY	INTERNET SOFTWARE AND SERVICES	United States	27.33B	7.09146579484146	25.24

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2014-01-01

No.	Ticker	Company	Position	Sector	Industry	Country	Market Cap	P/E	Price
1	FB	company name n/a		INFORMATION TECHNOLOGY	INTERNET SOFTWARE AND SERVICES	United States	91.64B	86.3086820938818	50.42
2	REGN	company name n/a		HEALTH CARE	BIOTECHNOLOGY	United States	22.07B	26.9012453851034	230.14
3	CELG	company name n/a		HEALTH CARE	BIOTECHNOLOGY	United States	64.65B	43.3263284868759	157.2
4	DAL	company name n/a		INDUSTRIALS	AIRLINES	United States	20.77B	10.0100692093050	24.21

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2014-04-01

No.	Ticker	Company	Position	Sector	Industry	Country	Market Cap	P/E	Price
1	MU	company name n/a		INFORMATION TECHNOLOGY	SEMICONDUCTORS AND SEMICONDUCTOR EQUIPMENT	United States	22.90B	17.9469744918987	21.66
2	FB	company name n/a		INFORMATION TECHNOLOGY	INTERNET SOFTWARE AND SERVICES	United States	106.01B	69.0693075123794	54.71
3	REGN	company name n/a		HEALTH CARE	BIOTECHNOLOGY	United States	22.07B	51.4694418228505	230.14
4	NOC	company name n/a		INDUSTRIALS	AEROSPACE AND DEFENSE	United States	25.14B	13.3672482370959	113.23

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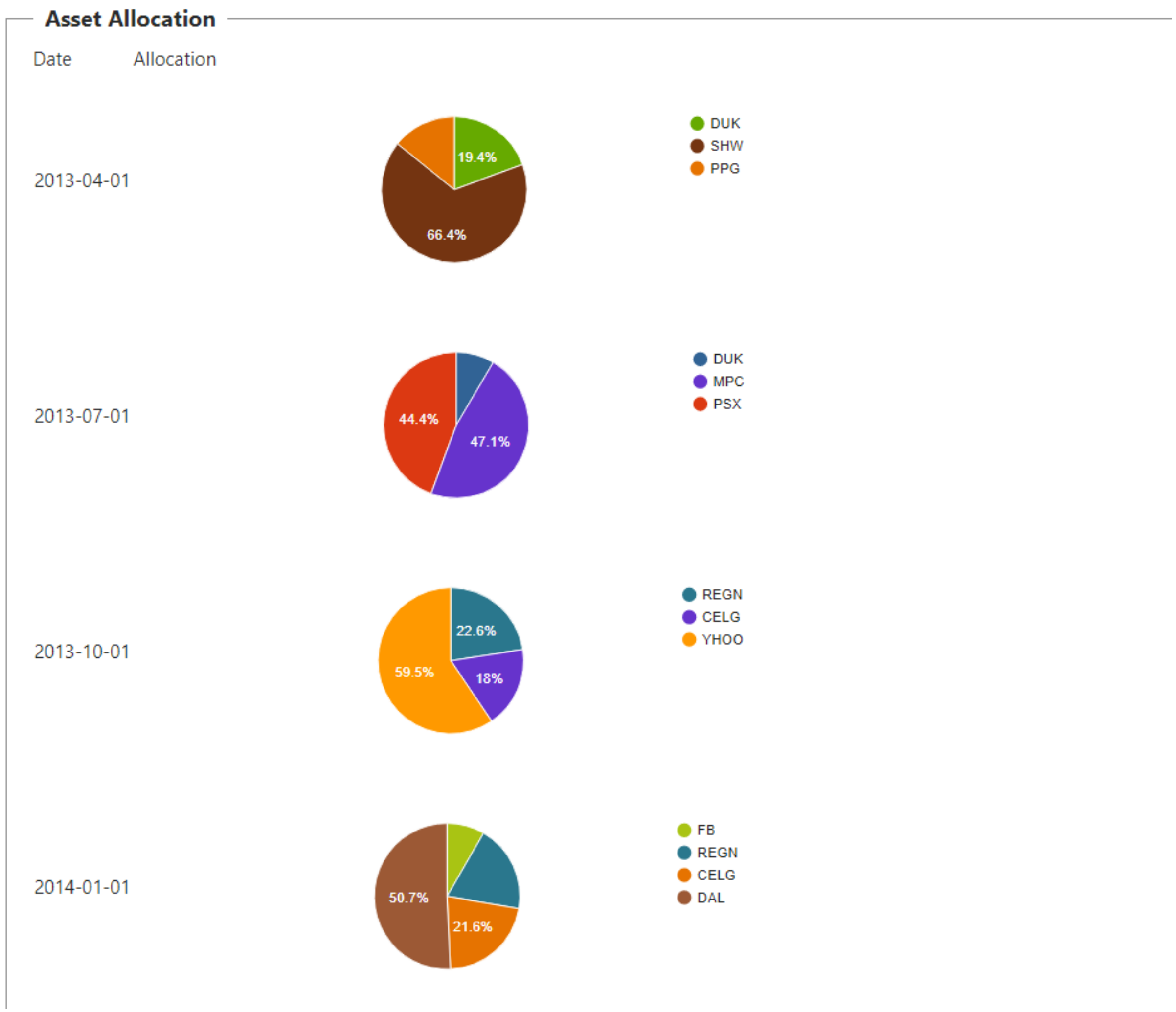
2014-07-01

No.	Ticker	Company	Position	Sector	Industry	Country	Market Cap	P/E	Price
1	FRX	company name n/a		HEALTH CARE	PHARMACEUTICALS	United States	25.29B	149.611211322777	93.31
2	FB	company name n/a		INFORMATION TECHNOLOGY	INTERNET SOFTWARE AND SERVICES	United States	124.68B	63.7189822000766	62.62
3	PCLN	company name n/a		CONSUMER DISCRETIONARY	INTERNET AND DIRECT MARKETING RETAIL	United States	65.25B	32.8185066908094	1251.37

NM FinTech – Capital Allocation

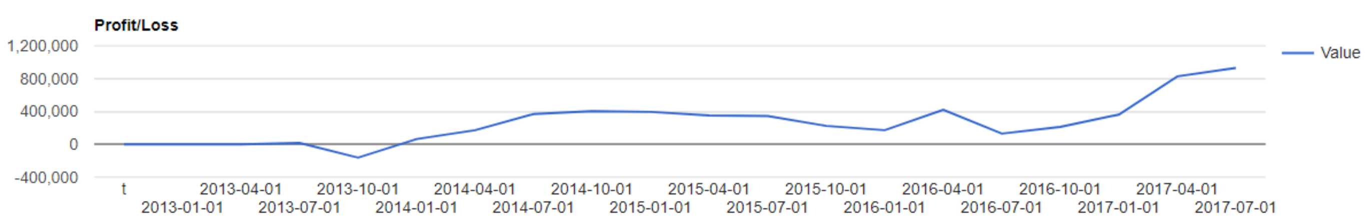


NM FinTech – Performance Statistics

Performance Measures

Measure	Value
ProfitLoss	928480.8442017422
ProfitAfterTransactionFee (TransactionFeeByPercentage (0.002500))	848722.0456584138
TransactionFeeByPercentage (0.002500)	79758.79854332838
average annual rate of return (capital(1000000.0), interval(2013-01-01T00:00:00.000-05:00/2017-07-04T00:00:00.000-04:00))	0.14036286355414584
CommissionProfitRatio (Commission (0.250000%))	0.08590247073099358
MaxDrawdown	289679.4941723208
Max Drawdown Percentage (capital(1000000.000000))	0.20391959992095543
MaxDrawdownDuration(unit(PT86400S))	457.0416666666667
beta (interval(2013-01-01T00:00:00.000-05:00/2017-07-04T00:00:00.000-04:00), period(P1Y), capital(1000000.000000))	0.24737510534676904
alpha (interval(2013-01-01T00:00:00.000-05:00/2017-07-04T00:00:00.000-04:00), period(P1Y), capital(1000000.000000))	0.17003837623625964
Information Ratio For Periods (capital(1000000.000000), benchmark(0.000000), period(P1Y))	0.8630375544288351
OmegaForPeriods(capital(1000000.000000)threshold(0.000000)period(P1Y))	Infinity
Sortino ratio (target(0.000000), capital(1000000.000000), interval(2013-01-01T00:00:00.000-05:00/2017-07-04T00:00:00.000-04:00), period(P1Y)	1.9723012874804964
Calmar ratio (capital(1000000.000000), interval(2013-01-01T00:00:00.000-05:00/2017-07-04T00:00:00.000-04:00)	0.6883245338287941
Execution Count	101

P&L



NM FinTech – By-Period Performance