Algorithmic Trading as a Science

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Speaker Profile

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- Quantitative Trader
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Definition

- Quantitative trading is the systematic execution of trading orders decided by quantitative market models.
- It is an arms race to build
  - more comprehensive and accurate prediction models (mathematics)
  - more reliable and faster execution platforms (computer science)

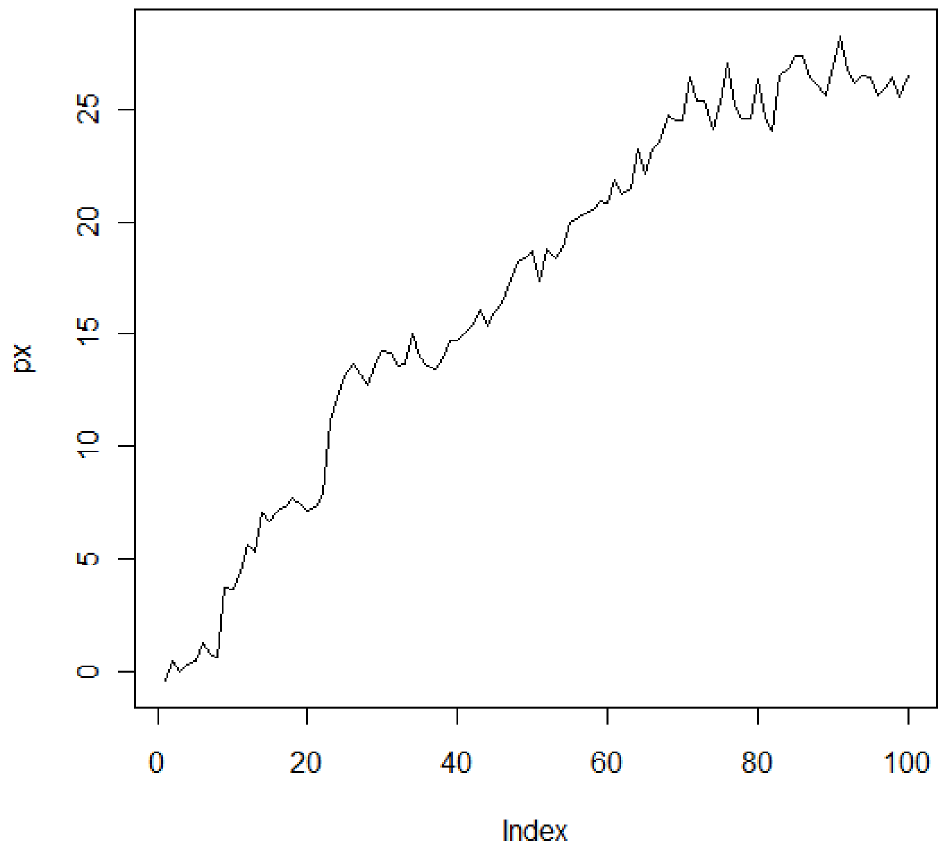
Scientific Trading Models

- Scientific trading models are supported by logical arguments.
  - can list out assumptions
  - can quantify models from assumptions
  - can deduce properties from models
  - can test properties
  - can do iterative improvements

Superstition

- Many “quantitative” models are just superstitions supported by fallacies and wishful-thinking.

Let's Play a Game



Impostor Quant. Trader

- Decide that this is a bull market
  - by drawing a line
  - by (spurious) linear regression
- Conclude that
  - the slope is positive
  - the t-stat is significant
- Long
- Take profit at 2 upper sigmas
- Stop-loss at 2 lower sigmas

Reality

- r = rnorm(100)
- px = cumsum(r)
- plot(px, type='l')

Mistakes

- Data snooping
- Inappropriate use of mathematics
  - assumptions of linear regression
    - linearity
    - homoscedasticity
    - independence
    - normality
- Ad-hoc take profit and stop-loss
  - why 2?
- How do you know when the model is invalidated?

Fake Quantitative Models

- Assumptions cannot be quantified
- No model validation against the current regime
- Cannot explain winning and losing trades
- *Cannot be analyzed (systematically)*

Extensions of a Wrong Model

- Some traders elaborate on this idea by
  - using a moving calibration window (e.g., Bands)
  - using various sorts of moving averages (e.g., MA, WMA, EWMA)

A Scientific Approach

- Start with a market insight (hypothesis)
  - hopefully without peeking at the data
- Translate English into mathematics
  - write down the idea in math formulae
- In-sample calibration; out-sample backtesting
- Understand why the models work or fail
  - in terms of model parameters
  - e.g., unstable parameters, small p-values

MANY Mathematical Tools Available

- Markov model
- co-integration
- stationarity
- hypothesis testing
- bootstrapping
- signal processing, e.g., Kalman filter
- returns distribution after news/shocks
- time series modeling
- The list goes on and on……

A Sample Trading Idea

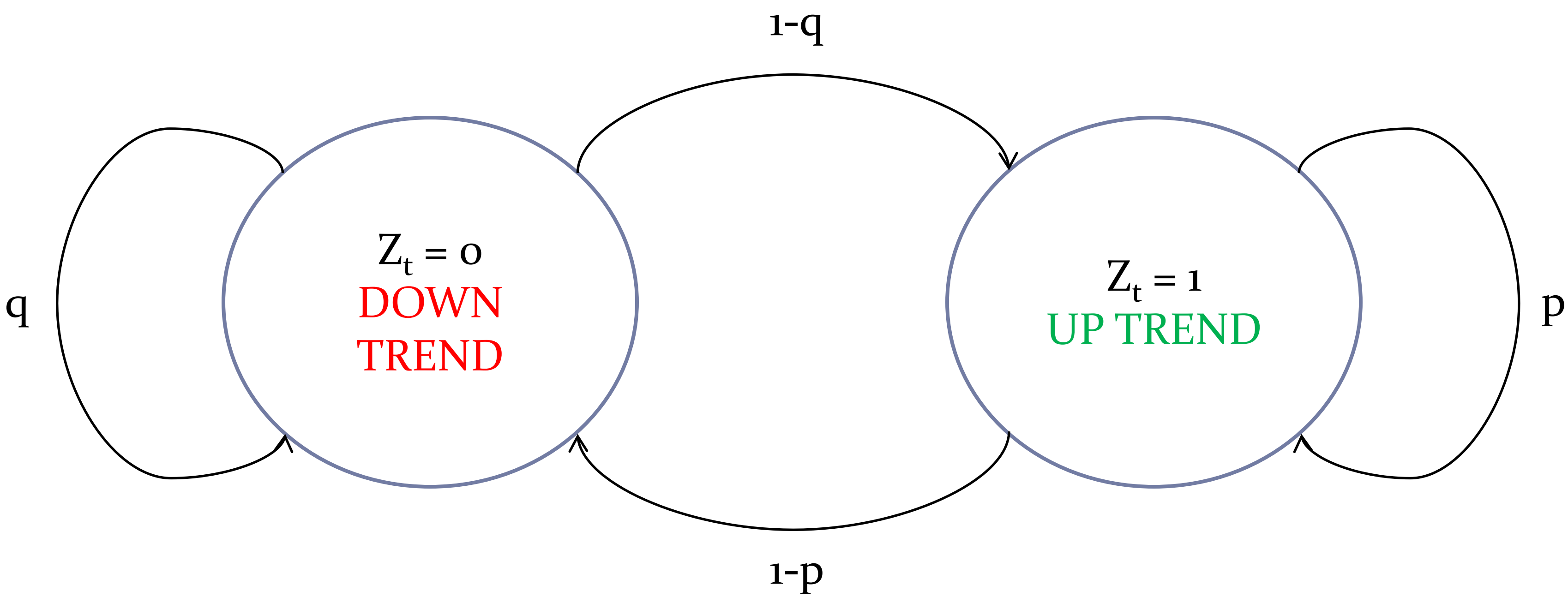
- When the price trends up, we buy.
- When the price trends down, we sell.

What is a Trend?

An Upward Trend

- More positive returns than negative ones.
- Positive returns are persistent.

Knight-Satchell-Tran Z_t



Knight-Satchell-Tran Process

- $R_t = \mu_l + Z_t \varepsilon_t - (1 - Z_t)\delta_t$
  - μ_l: long term mean of returns, e.g., 0
  - ε_t, δ_t: positive and negative shocks, non-negative, i.i.d
- $f_\varepsilon(x) = \dfrac{{\lambda_1}^{\alpha_1} x^{\alpha_1 - 1}}{\Gamma(\alpha_1)} e^{-\lambda_1 x}$
- $f_\delta(x) = \dfrac{{\lambda_2}^{\alpha_2} x^{\alpha_2 - 1}}{\Gamma(\alpha_2)} e^{-\lambda_2 x}$

How Signal Do We Use?

- Let’s try Moving Average Crossover.

Moving Average Crossover

- Two moving averages: slow (n) and fast (m).
- Monitor the crossovers.
- $B_t = \left(\frac{1}{m}\sum_{j=0}^{m-1} P_{t-j}\right) - \left(\frac{1}{n}\sum_{j=0}^{n-1} P_{t-j}\right), n > m$
- Long when $B_t \geq 0$.
- Short when $B_t < 0$.

How to choose n and m?

- For most traders, it is an art (guess), not a science.
- Let's make our life easier by fixing $m = 1$.
  - Why?

GMA(n , 1)

- $B_t \geq 0$ iff $P_t \geq \left(\prod_{j=0}^{n-1} P_{t-j}\right)^{\frac{1}{n}}$
  - $R_t \geq -\sum_{j=1}^{n-2} \frac{n-(j+1)}{n-1} R_{t-j}$ (by taking log)
- $B_t < 0$ iff $P_t < \left(\prod_{j=0}^{n-1} P_{t-j}\right)^{\frac{1}{n}}$
  - $R_t < -\sum_{j=1}^{n-2} \frac{n-(j+1)}{n-1} R_{t-j}$ (by taking log)

What is n?

- $n = 2$
- $n = \infty$

GMA(2, 1)

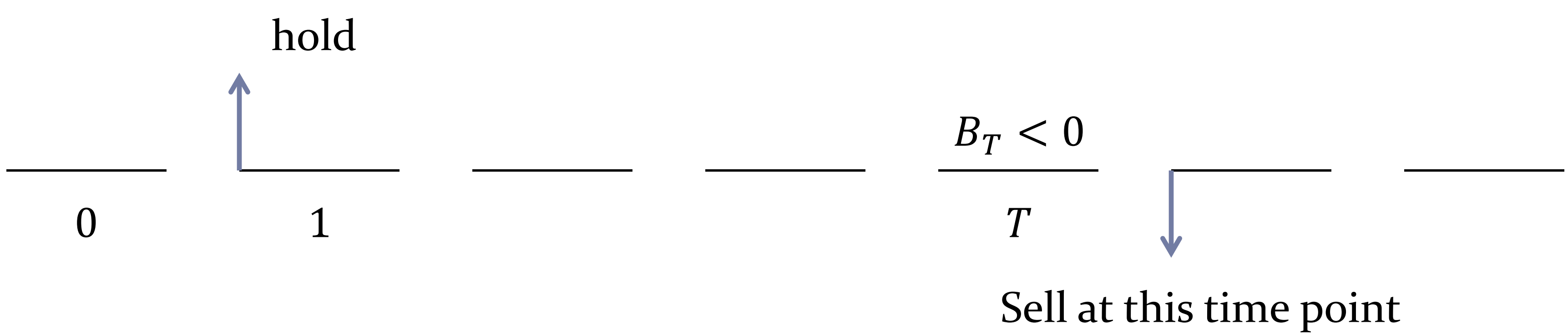
- Assume the long term mean is 0, $\mu_l = 0$.
- $(B_t \geq 0) \equiv (R_t \geq 0) \equiv (Z_t = 1)$
- $(B_t < 0) \equiv (R_t < 0) \equiv (Z_t = 0)$

Naïve MA Trading Rule

- Buy when the asset return in the present period is positive.
- Sell when the asset return in the present period is negative.

How Much Money Will I Make?

- T Period Return:
  - $RR_T = \sum_{t=1}^{T} R_t \times I_{\{B_{t-1} \geq 0\}}$



Expected Holding Time

- $P(N = T)$
- $= P(B_T < 0, B_{T-1} \geq 0, \dots, B_1 \geq 0, B_0 \geq 0)$
- $= P(Z_T = 0, Z_{T-1} = 1, \dots, Z_1 = 1, Z_0 = 1)$
- $= P(Z_T = 0, Z_{T-1} = 1, \dots, Z_1 = 1 | Z_0 = 1) P(Z_0 = 1)$
- $= \begin{cases} \Pi p^{T-1}(1-p), & \mathrm{T} \geq 1 \\ 1 - \Pi, & \mathrm{T} = 0 \end{cases}$

- Stationary probabilities
  - $\Pi = \frac{1-q}{2-p-q}$

My Returns Distribution (1)

- $\Phi_{RR_T|N=T}(s)$
- $= \mathrm{E}\left[e^{\left\{i\left[\sum_{t=1}^{T} R_t \times I_{\{B_{t-1} \geq 0\}}\right]s\right\}} | N = T\right]$
- $= \mathrm{E}\left[e^{\left\{i\left[\sum_{t=1}^{T} R_t \times I_{\{B_{t-1} \geq 0\}}\right]s\right\}} | B_T < 0, B_{T-1} \geq 0, \dots, B_0 \geq 0\right]$
- $= \mathrm{E}\left[e^{\left\{i\left[\sum_{t=1}^{T} R_t\right]s\right\}} | Z_T = 0, Z_{T-1} = 1, \dots, Z_1 = 1\right]$
- $= \mathrm{E}\left[e^{\{i[\varepsilon_1 + \cdots + \varepsilon_{T-1} - \delta_T]s\}}\right]$
- $= \begin{cases} {\Phi_\varepsilon}^{T-1}(s)\Phi_\delta(-s), & \mathrm{T} \geq 1 \\ \Phi_\delta(-s), & \mathrm{T} = 0 \end{cases}$

My Returns Distribution (2)

- $\Phi_{RR_T}(s) = \sum_{T=0}^{\infty} \mathrm{E}\left[e^{\left\{i\left[\sum_{t=1}^{T} R_t \times I_{\{B_{t-1} \geq 0\}}\right]s\right\}} | N = T\right] P(N = T)$
- $= \sum_{T=1}^{\infty} \Pi p^{T-1}(1-p)\Phi_{\varepsilon}{}^{T-1}(s)\Phi_{\delta}(-s) + (1-\Pi)\Phi_{\delta}(-s)$
- $= (1-\Pi)\Phi_{\delta}(-s) + \Pi(1-p)\frac{\Phi_{\delta}(-s)}{1-p\Phi_{\varepsilon}(s)}$

Expected P&L

- $\mathrm{E}(RR_T) = -i{\Phi_{RR_T}}'(0)$
- $= \frac{1}{1-p}\{\Pi p\mu_\varepsilon - (1-p)\mu_\delta\}$

When Will My Strategy Make Money?

- The expected return is positive when
  - $\mu_\varepsilon \geq \frac{1-p}{\Pi p}\mu_\delta$, shock impact
  - $\mu_\varepsilon \gg \mu_\delta$, shock impact
  - $\Pi p \geq 1 - p$, if $\mu_\varepsilon \approx \mu_\delta$, persistence

What About GMA(∞,1)

- Repeat the steps above.
- $\mathrm{E}(RR_T) = -[1 - p(1 - \Pi)][\mu_\varepsilon + \mu_\delta]$

When Will GMA(∞,1) Make Money?

Model Benefits (1)

- It makes “predictions” about which regime we are now in.
- We quantify how useful the model is by
  - the parameter sensitivity
  - the duration we stay in each regime
  - the state differentiation power

Model Benefits (2)

- We can explain winning and losing trades.
  - Is it because of calibration?
  - Is it because of state prediction?
- We can deduce the model properties.
  - Are 2 states sufficient?
  - prediction variance?
- We can justify take-profit and stop-loss based on trader utility function.

Backtesting

- Backtesting simulates a strategy (model) using historical or fake (controlled) data.
- It gives an idea of how a strategy would work in the past.
  - It does not tell whether it will work in the future.
- It gives an objective way to measure strategy performance.
- It generates data and statistics that allow further analysis, investigation and refinement.
  - e.g., winning and losing trades, returns distribution
- It helps choose take-profit and stop-loss.

Some Performance Statistics

- p&l
- mean, stdev, corr
- Sharpe ratio
- confidence intervals
- max drawdown
- breakeven ratio
- biggest winner/loser
- breakeven bid/ask
- slippage

Omega

- $\Omega(L) = \frac{\int_L^b [1 - F(x)]dx}{\int_a^L F(x)dx} = \frac{C(L)}{P(L)}$
- The higher the ratio, the better
- Ratio of the probability of having a gain by the probability of having a loss
- Do not assume Normality
- Use the whole returns distribution

Performance on MSCI Singapore

Bootstrapping

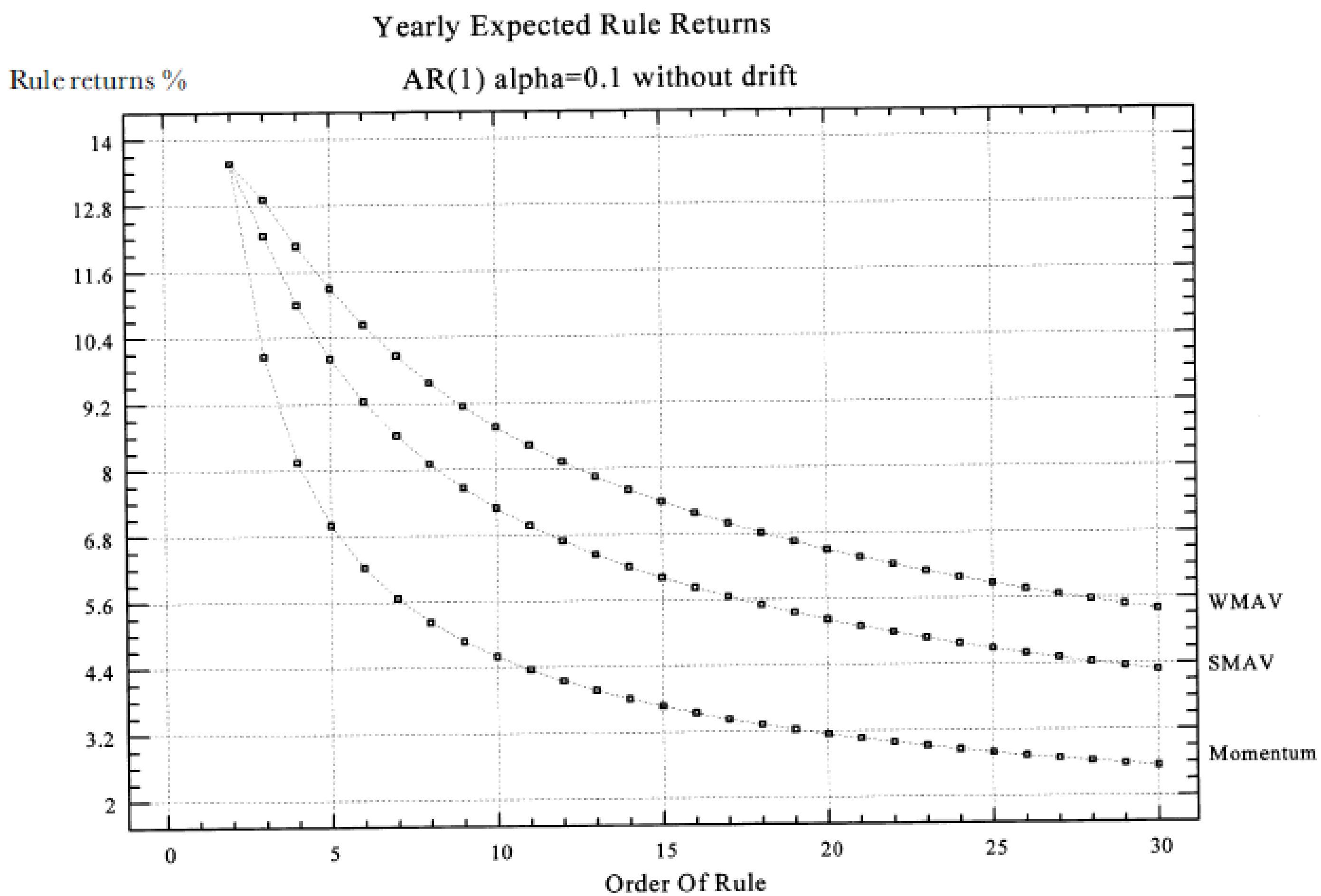
- We observe only one history.
- What if the world had evolve different?
- Simulate "similar" histories to get confidence interval.
- White's reality check (White, H. 2000).

Fake Data

Returns: AR(1)

- $X_t = \alpha X_{t-1} + \varepsilon_t$
  - Auto-correlation is required to be profitable.
  - The smaller the order, the better. (quicker response)

Returns: AR(1)



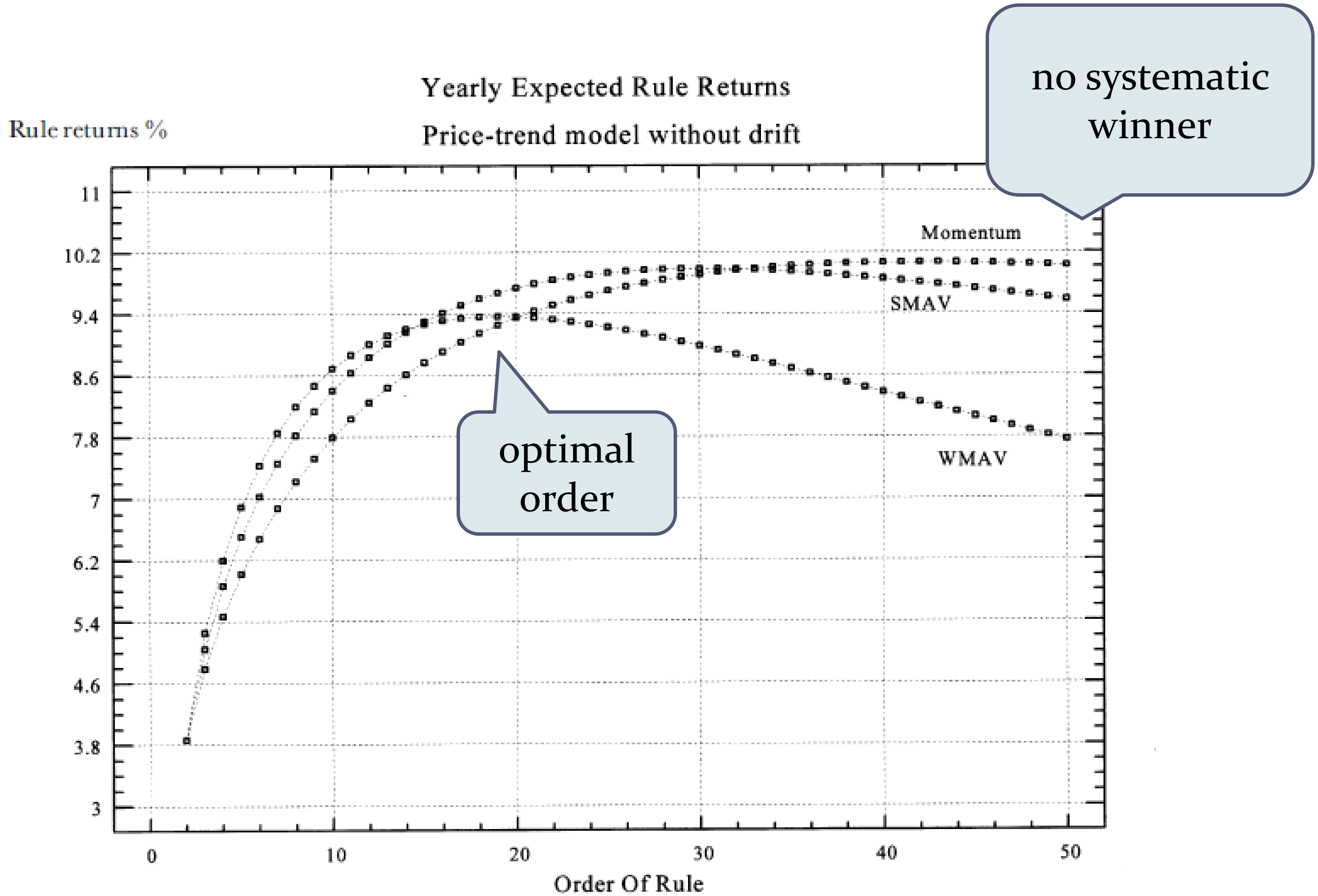
Yearly Expected Rule Returns
AR(1) alpha=0.1 without drift
Rule returns %
14
12.8
11.6
10.4
9.2
8
6.8
5.6
4.4
3.2
2
0
5
10
15
20
25
30
Order Of Rule
WMAV
SMAV
Momentum

Returns: ARMA(1, 1)

AR MA

- $(X_t - \mu) - p(X_{t-1} - \mu) = \varepsilon_t - q\varepsilon_{t-1}$
- Prices tend to move in one direction (trend) for a period of time and then change in a random and unpredictable fashion.

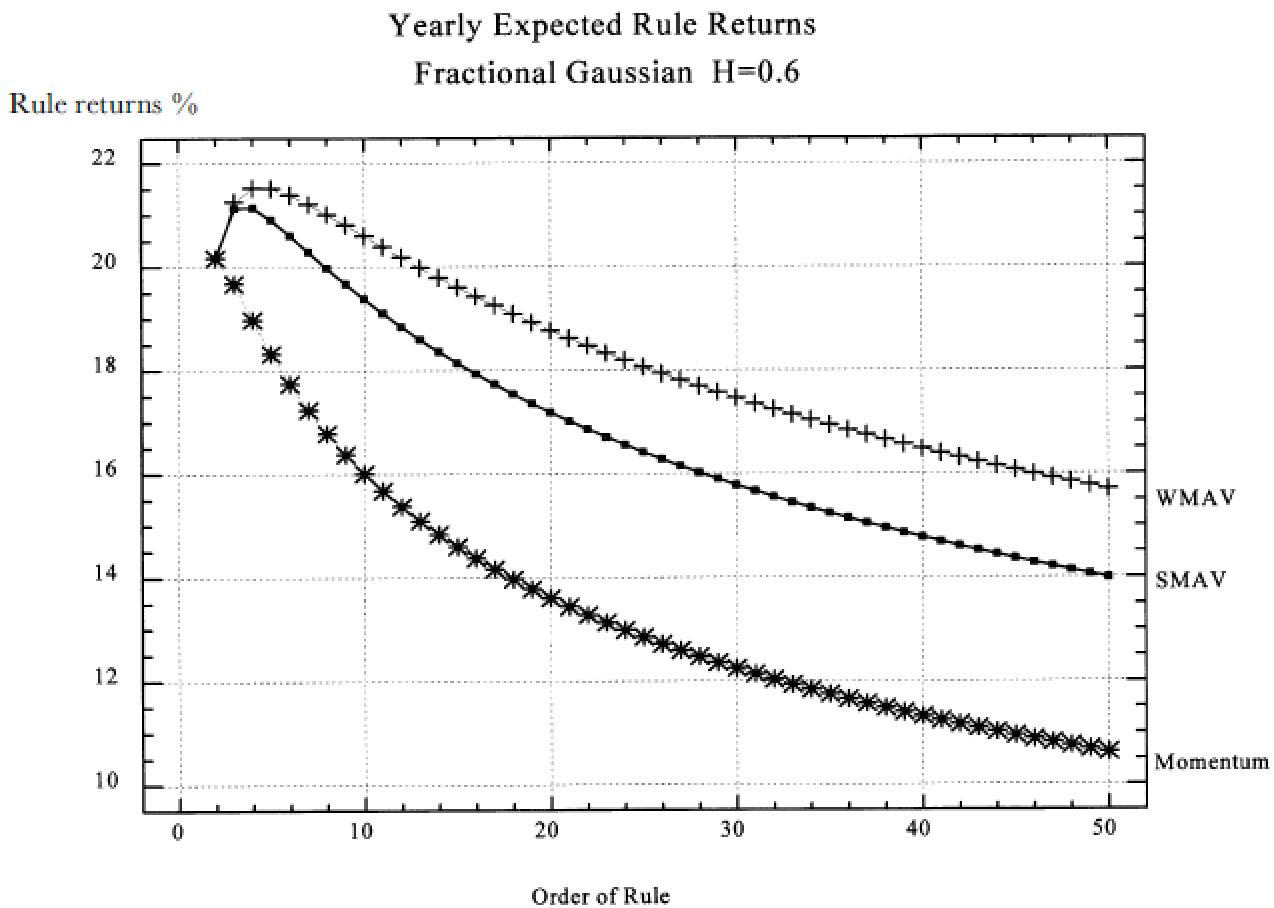
Returns: ARMA(1, 1)



Returns: ARIMA(0, d, 0)

- $\nabla^d (X_t - \mu) = e_t$
- Irregular, erratic, aperiodic cycles.

Returns: ARIMA(0, d, 0)



ARCH + GARCH

- The presence of conditional heteroskedasticity, if unrelated to serial dependencies, may be neither a source of profits nor losses for linear rules.

A good Backtester (1)

- allow easy strategy programming
- allow plug-and-play multiple strategies
- simulate using historical data
- simulate using fake, artificial data
- allow controlled experiments
  - e.g., bid/ask, execution assumptions, news

A good Backtester (2)

- generate standard and user customized statistics
- have information other than prices
  - e.g., macro data, news and announcements
- Auto calibration
- Sensitivity analysis
- Quick

Matlab/R

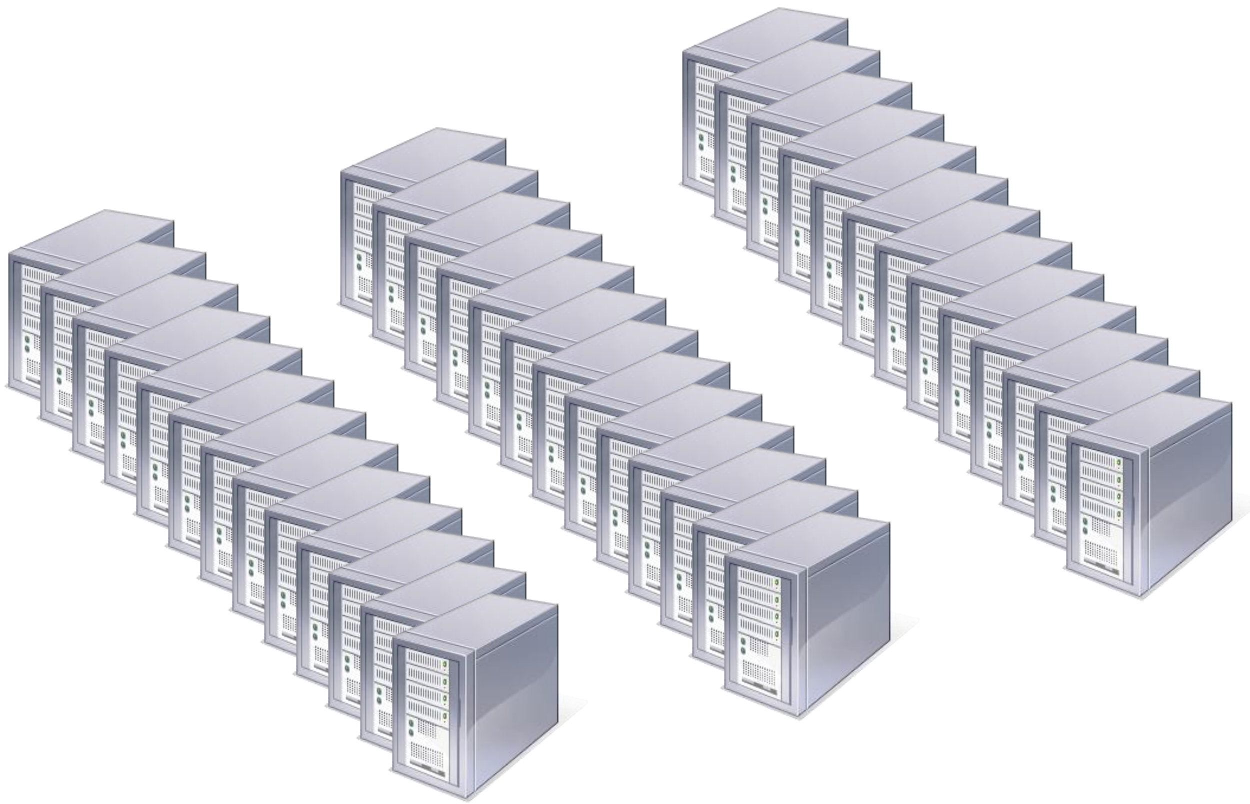
- They are very slow. These scripting languages are interpreted line-by-line. They are not built for parallel computing.
- They do not handle a lot of data well. How do you handle two year worth of EUR/USD tick by tick data in Matlab/R?
- There is no modern software engineering tools built for Matlab/R. How do you know your code is correct?
- The code cannot be debugged easily. Ok. Matlab comes with a toy debugger somewhat better than gdb. It does not compare to NetBeans, Eclipse or IntelliJ IDEA.

Calibration

- Most strategies require calibration to update parameters for the current trading regime.
- Occam's razor: the fewer parameters the better.
- For strategies that take parameters from the Real line: Nelder-Mead, BFGS
- For strategies that take integers: Mixed-integer non-linear programming (branch-and-bound, outer-approximation)

Global Optimization Methods

$$f$$



Sensitivity

- How much does the performance change for a small change in parameters?
- Avoid the optimized parameters merely being statistical artifacts.
- A plot of measure vs. d(parameter) is a good visual aid to determine robustness.
- We look for plateaus.

Iterative Refinement

- Backtesting generates a large amount of statistics and data for model analysis.
- We may improve the model by
  - regress the winning/losing trades with factors
  - identify, delete/add (in)significant factors
  - check serial correlation among returns
  - check model correlations
  - the list goes on and on……

Implementation

- Connectivity to exchanges
  - e.g., ION, RTS
- Platform dependent APIs
- Programming languages
  - Java, C++, C#, VBA, Matlab

Summary

- Market understanding gives you an intuition to a trading strategy.
- Mathematics is the tool that makes your intuition concrete and precise.
- Programming is the skill that turns ideas and equations into reality.